Digital Elevation Models of Axel Heiberg Island Glaciers

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Summary

We describe two digital elevation models (DEMs) of the extensively glacierized Expedition Fiord area of central Axel Heiberg Island, Nunavut, Canada. Both derive from estimates made by eye on large-scale maps. The White Glacier (WG) DEM, with 50 m resolution, is based on a 1:10,000 scale map and aerial photographs dating from 1960. The Thompson Glacier (TG) DEM, with 100 m resolution, is based mainly on a 1:50,000 scale map and on 1959 aerial photographs. Part of it derives from a 1:100,000 scale map, from which many of the elevations were estimated by a spatial interpolation algorithm.

The Expedition Fiord area is well mapped by comparison with the rest of the Canadian High Arctic, for most of which the largest scale of map coverage is 1:250,000. We compare the Expedition Fiord DEMs with each other and with the CDED DEM, based on the 1:250,000 scale map of the area, and with the GLOBE DEM, based on a 1:1,000,000 scale map. The resolution of CDED is 92 m × 34 m (meridional × zonal). The resolution of GLOBE is 930 m × 170 m. We present a procedure for converting the local coordinates of the Expedition Fiord maps to transverse Mercator coordinates. This procedure gives good accuracy, although the derivation of the local coordinate system remains somewhat mysterious.

Replicate readings by multiple map readers allow us to estimate the map-reading error for the large-scale DEMs. The errors are reduced markedly by proofreading. The differences between replicate readings are unbiased. For the WG DEM, the random map-reading error is of the order of ±1 m. Analysis of the frequency distributions and the spatial autocorrelation of differences shows that they are not normally distributed. They are also moderately correlated, with decorrelation distances of 1-3 DEM resolution elements. That is, map-reading errors tend to occur in clusters.

For the TG DEM the map-reading error is of the order of 4-8 m. The greater error is due partly to less thorough proofreading, but mainly to the greater contour interval, 25 m as opposed to 10 m on the WG map. Replicate readings from the two maps have root-mean-square differences of about 20 m, a figure which shrinks to 12 m when gross errors are identified and excluded. Partitioning this mapping error between the two maps requires an assumption about which, if either, of them is the truth. Depending on this assumption, the total error in the WG DEM (the map-reading error and some share of the mapping error, added in quadrature) may range from its map-reading error, about 1 m, up to about 19 m, while for the TG DEM the total error may reach 20 m. Excluding gross errors, the maximum estimates of total error decrease to 11 m for the WG DEM and 12 m for the TG DEM.

Thus we have identified two obstacles to formal description of the DEM errors. First, there is no objective basis for partitioning the mapping error between the two maps. Second, the gross errors cannot be accommodated satisfactorily. A third obstacle is that the errors are only nominally “one-sigma” errors. Because the usual statistical assumptions are violated, the errors define confidence regions narrower than the usual 68% by some unknown amount.

The spatial interpolation algorithm used for the 1:100,000-scale portion of the TG DEM performs well. Comparisons with visual estimates suggest a map-reading error of 13 m and a total error of 13-24 m.

Comparisons between TG and the smaller-scale CDED and GLOBE DEMs show that the latter are significantly more uncertain. Total error is estimated as 90 m for CDED and 158 m for GLOBE.

Each of the DEMs contains artefacts related to the contour interval of its parent map. These artefacts, while they look disconcerting, appear to make only a modest contribution to map-reading error.

The available evidence suggests that, in comparison with mapping error, map-reading error is small but not necessarily negligible. That is, the DEMs are good to very good representations of the information in the maps. The maps, however, are less good representations of the terrain than the DEMs are of the maps.

Errors in slopes calculated from the DEM increase as map scale decreases. For example, if the WG DEM represents truth, the uncertainty in slope estimates is about 2° for TG, 5° for CDED, and 9° for GLOBE. There is some evidence that map-reading errors make a proportionally greater contribution to total error for slopes. A more clear-cut finding is that maps of larger scale have steeper slopes. Smaller scale implies more
generalization of the terrain. It is not clear, however, how this bias might be corrected in the smaller-scale DEMs. Differences in the frequency distribution of slopes are such that a simple additive correction would reduce, not increase, the resemblance of the smaller-scale DEMs to the larger-scale DEMs.

It has not been possible through this analysis to specify elevation errors rigorously, but the errors are well correlated with the contour interval of the parent map, appearing to be roughly equal to one half of the contour interval. There is no obvious reason why this should be so, but as a rule of thumb for practical contexts it should work well.
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INTRODUCTION

This note documents the sources and means of preparation of two digital elevation models (DEMs) of the Expedition Fiord area of central Axel Heiberg Island, Nunavut, Canada (Figure 1). The less extensive of the two DEMs represents White Glacier, a valley glacier which has been the subject of glaciological attention since 1959; the glacierized area is about 44 km$^2$. The more extensive DEM covers the Expedition Fiord glacier complex, consisting of those glacierized catchments which are hydrologically tributary to the head of Expedition Fiord; not including peripheral ice, the glacierized area is about 580 km$^2$. A detailed analysis of errors is provided and the DEMs are compared with two other published DEMs based on small-scale topographic maps.

Figure 1. Axel Heiberg Island in the context of the Queen Elizabeth Islands (inset). Glaciers are shaded. The rectangle in the centre of the island encloses the domain of the Thompson Glacier DEM. Exp: Expedition Fiord. E: Eureka. Universal Transverse Mercator projection, zone 15.

When the White Glacier (WG) DEM was completed in the early 1990s, DEMs were still not the commonplace tools they have now become in the earth sciences. Even then, however, the method adopted to prepare the DEM was old-fashioned: elevations were read by eye from maps and the electronic product was generated by keyboarding. Essentially the same method, with refinements, was used for the Thompson Glacier (TG) DEM, which was completed only recently. This brute-force approach was adopted in part to tap a source of inexpensive student labour. In addition to money, the students gained valuable experience in data collection and validation, and they have made in sum an important contribution to the development of a modern infrastructure for glaciological and other research in central Axel Heiberg Island.
The brute-force approach has another advantage. Repeated reading of the same map by different workers, and of different maps where they overlap, provides a natural way to investigate and quantify errors. It turns out that the method is very accurate, and it is certainly competitive with the automated methods of DEM production which have evolved so rapidly in recent years.

The next section summarizes the cartographic history of central Axel Heiberg Island, focussing on the maps which were the sources of the two DEMs. Section three provides necessary geodetic background and discusses the geodetic and cartographic uncertainties underlying the DEMs. The next two sections examine respectively the WG DEM and the TG DEM, describing how they were prepared and seeking to provide as complete an analysis of errors as is permitted by the available information. The sixth section focusses on external comparisons between the Expedition Fiord DEMs and two smaller-scale DEMs, CDED and GLOBE. The final section attempts a synthesis.
CARTOGRAPHIC HISTORY

Explorers’ sketches aside (Sverdrup 1904), Expedition Fiord, then called South Fiord, first appeared on a topographic map in 1948. This map, at a scale of 1:250,000, was drawn from oblique aerial photographs taken in 1947 and 1948. It is documented by Ommanney (1969a,b), together with later maps from similar material at scales of 1:506,880 and 1:500,000. All of these maps were highly generalized. In 1958 and 1959 vertical aerial photography of Axel Heiberg Island was completed, at a typical scale of 1:60,000, and these photographs remain the principal source for detailed cartographic coverage of the island. The 1:250,000 scale maps prepared from them were published in 1966 and 1967, that for the Expedition Fiord area having the name Strand Fiord and the National Topographic Series (NTS) number 59H (Department of Energy, Mines and Resources 1967). Meanwhile the Jacobsen-McGill Arctic Research Expedition had established the McGill Arctic Research Station at Colour Lake near the terminus of Thompson Glacier. This scientific venture began in 1960 (B.S. Müller 1961; Müller et al. 1963), after a reconnaissance in 1959. It had broad interests across the natural sciences with a concentration on glaciology. Among its early activities were special-purpose aerial photography and a detailed ground survey of the region at the head of Expedition Fiord (Haumann 1961, 1963).

These resources (Müller 1963a) were the basis for publication of a number of maps at scales as large as 1:2,500 (e.g. National Research Council 1962b,c, 1965b, 1966; Arnold 1981). The special challenges of glaciological cartography are discussed in a number of publications by participants in the expedition (Blachut 1961a, 1963a,b; Blachut and Müller 1966; McKortel 1963; Müller 1963b). At about the same time, large-scale maps of glaciers in the Canadian High Arctic were also published by Arnold (1966; Meighen Ice Cap, Meighen Island) and Konecny (1966; Per Ardua Glacier, northern Ellesmere Island). These apart, however, and thanks to the Jacobsen-McGill Arctic Research Expedition, the Expedition Fiord area is still one of the few glacierized parts of the Canadian High Arctic for which large-scale maps are available. Only a small number of NTS sheets have been published at 1:50,000 scale.

Ommanney (1987) gives an exhaustive bibliography of the expedition in general and of its maps in particular. An updated bibliography of glaciological studies on Axel Heiberg Island is given by Cogley (1999b). Photographic coverage is described by Ommanney (1969a,b) and Cogley and Adams (2000). The reports by Ommanney, and also Dunbar and Greenaway (1956), have abundant samples of the early oblique photography of Axel Heiberg Island flown between 1947 and 1953. The only recent printed cartographic product is a large-scale orthophotomap of the terminus of Thompson Glacier (Institute of Cartography 1998) based on 1977 photographs.

Three of the expedition’s maps are the focus of the present work. The maps which make best use of the ground survey are those at scales of 1:50,000, covering the Thompson Glacier region (National Research Council 1962a), and 1:10,000, covering White Glacier in two sheets (National Research Council 1965a). There is also a 1:100,000 scale map covering a greater area (McGill University 1963), including a substantial part of Müller Ice Cap, but outside the region of the 1:50,000 scale map the geodetic control is loose to non-existent. This map is referred to here as the EF map.

Digital cartographic products have recently become available. Those of most relevance here are the vector files published as part of NTDB, the National Topographic Data Base (Centre for Topographic Information 1999) and, more directly, the digital elevation model CDED (Centre for Topographic Information 2000). These are digital renditions of the NTS 1:250,000 scale maps. The CDED DEM has meridional resolution of 3″ (3 arc seconds) and zonal resolution of 6″-12″, or 92 m by 32 to 64 m. At a still smaller scale the Digital Chart of the World (Defense Mapping Agency 1992) and the related GLOBE DEM (Hastings et al. 1999) cover Axel Heiberg Island at useful resolution (30″ for GLOBE, or meridional and zonal resolutions of 930 m and 170 m). These two small-scale DEMs, CDED and GLOBE, are compared with the detailed local DEMs in a later section.

It is remarkable that, except for a few maps of glacier terminuses at very large scale, all of the standard cartographic material for Expedition Fiord represents the state of the terrain and the glaciers during the years from 1959 to about 1965. Imagery of various kinds is available over a broader span (Cogley and Adams
2000), and modern satellite sensors offer considerable scope for the investigation of change in the landscape. Cogley et al. (1996b), for example, used aerial photographs and synthetic aperture radar to reconstruct the positions of glacier terminuses at dates between 1948 and 1995. L. Copland of the University of Alberta (personal communication, 2001) has procured, through the EROS Data Center of the U.S. Geological Survey, an Aster DEM of the Middle Glacier area some 80 km northwest of Expedition Fiord. The Aster sensor, which became operational in 2000 on board the EOS Terra satellite, has a stereoscopic nadir- and aft-viewing mode with horizontal resolution of 15 m, and the Aster processing system is capable of producing either uncontrolled or ground-controlled estimates of surface elevation from such stereo pairs. The Middle Glacier DEM, which is uncontrolled and was constructed from Level 1A imagery, is best regarded as an interesting proof of concept. It is clear, however, that mapping from space promises to revolutionize the quantitative monitoring of glaciological change.

Considered as physical objects, some of the Jacobsen-McGill Expedition maps are now fragile and some have become rare. Budkewitsch (2002) has scanned several of the maps (McGill University 1963; National Research Council 1962a, 1962b, 1962c, 1965a, 1965b) and they are available from him in the form of TIFF files on CD-ROMs. The present work may be seen as a similar exercise in data rescue in support of analyses by modern digital methods.
Digital Elevation Models of Axel Heiberg Island Glaciers

GEODETIC BACKGROUND

Surveying and Photography

The ground survey and photographic flight programme which were the basis for mapping are described by Haumann (1963; see also Haumann 1961). A total of 110 control points, including 23 points forming the primary triangulation net, was occupied with a Wild T2 theodolite, and a substantial amount of higher-order work was also completed. At each primary point observations were made of from 2 to 10 of the other primary points. The primary and some of the secondary control points are plotted on Figure 6 below.

The photographs from which the WG map was drawn were taken on 2 August 1960. The flying height varied between flightlines but was typically 3050 m. The photographs from which the TG map and EF map were drawn were part of the national coverage programme. They were taken on 28 July 1959 and 13 August 1959 from a height of 9000 m. The focal length of the camera was 153 mm for both 1960 and 1959 photography. Overlap between adjacent images was consistently 60%, so that a nominal figure for stereoscopic or x parallax at sea level (overlap times focal length) is 91.8 mm.

Positioning

Haumann (1963) and Blachut (1961b) describe the coordinate frame used for computations as a “local plane coordinate system”, giving no information about the chosen ellipsoid or projection. The horizontal datum is defined by measurements at the control points Astro 1 and Astro 2 (Figure 6). Observations of the Sun from Astro 1, with a Wild T2 theodolite equipped with a Roelofs prism, gave an azimuth of 35.17° for the baseline to Astro 2, the length of which was determined by tellurometer as 11740 m. These points have geographical coordinates (National Research Council 1962a) 90.7428056°W, 79.4100306°N (Astro 1) and 90.4119028°W, 79.4959750°N (Astro 2). The vertical datum is defined by the level of the sea near Level Point at 0100h UTC on 2 July 1960, Level Point itself being the reference bench mark (elevation 6.17 m). Haumann (1961) gives elevations of 277.51 m for Astro 1 and 609.16 m for Astro 2.

The grid lines on the various maps of the Expedition Fiord area are in local coordinates (referred to below as NRC coordinates), oriented to north by the Sun observations at Astro 1 and fixed to the Earth’s surface by assigning Astro 1 the arbitrary easting and northing (30000.00, 60000.00) in metres. Haumann (1961) gives the coordinates (36764.22, 69598.47) for Astro 2.

Essentially all of the available geodetic information is given above. It is not adequate for registering the Expedition Fiord maps to other published maps, or for that matter to any recognized ellipsoid. For this purpose, accordingly, an approximate procedure is needed.

First, we assume that the ellipsoid is the Clarke 1866 ellipsoid, which is part of the North American Datum 1927 (NAD27). This figure (e.g., Bomford 1971) has semimajor axis \( a_E = 6378206.4 \) m and eccentricity squared \( e^2 = 0.006768282 \).

We measured the grid positions of a set of twelve test points of “known” longitude and latitude using the meridian and parallel tick marks along the edges of the TG map. A transverse Mercator projection, with its central meridian at the given longitude of Astro 1, was able (after a translation to reconcile Mercator and NRC coordinates) to reproduce the grid measurements with reasonable accuracy. A range of central scale factors between 0.9990 and 1.0000 was tried and it was found that nothing was to be gained by departing from the value 0.9996 which is part of the Universal Transverse Mercator projection system.

Rms (root mean square) differences were 46.9 m in easting and 43.3 m in northing between the measured grid positions and those estimated using the projection. This is fair agreement, considering that the standard error of measurement on the map was not better than 10 m and the standard error of positioning (due to extending the short marginal tick marks with a straightedge) was at least 25–50 m. Further, until the implicit projection of the Expedition Fiord maps can be recovered more accurately through further investigation, this is the best available approximation. Although the positional errors are due largely and perhaps entirely to the map measurements, they are comparable to the 50 m resolution of the WG DEM and not much less
than the 100 m resolution of the TG DEM. They may therefore have a substantial influence on comparisons with other topographic maps and with satellite imagery, but because they are geodetic in character they are not of consequence for work relying solely on Expedition Fiord maps. Such work includes the comparisons described below between these maps and also, for example, the hypsometric analysis used by Cogley et al. (1996a) and Adams et al. (1998) and the photogrammetric analysis of Cogley et al. (1996b) and Cogley (1999a).

To summarize, measurements in NRC coordinates may be converted to geographical coordinates with good accuracy by

1) subtracting (30000.00, 60000.00) m, the NRC coordinates of Astro 1;
2) adding (500000.00, 8815604.36) m, the transverse Mercator coordinates of Astro 1 assuming the NAD27 datum;
3) and finally applying the inverse equations of the transverse Mercator projection.

The projection can be inverted with a standard UTM routine provided that the central longitude is specified as that of Astro 1, \(-90.7428056^\circ\). If it is desired to change from the NAD27 datum to another, such as WGS84, the geographical coordinates should be converted to geodetic cartesian coordinates, followed by the change of datum and a conversion from cartesian back to geographical coordinates.

**Contouring Errors**

Blachut and Müller (1966) suggest that the probable errors of the elevation $\Delta h$ and position $\Delta s$ of mapped contours on the WG map can be expressed respectively as

\[
\Delta h = p + q \tan \theta, \quad (1a)
\]
\[
\Delta s = q + p \cot \theta. \quad (1b)
\]

The two expressions are related by $\tan \theta = \Delta h / \Delta s$, where $\theta$ is the surface slope and $p = 0.2$ m and $q = 1.0$ m are empirical coefficients. Thus the vertical error is $\pm 1.2$ m for contours on slopes of 45° and approaches $\pm 0.2$ m as the slope approaches 0°. These are acknowledged to be best-case errors, field checking having revealed that actual errors are considerably larger. Haumann (1963) mentions elevation discrepancies as large as 4 m in mutual observations of mountain-peak control points with lines of sight well above the ground surface. A broader problem in glaciological cartography (e.g., Adalgeirsdóttir et al. 1998) is that uncertainty is particularly large in the accumulation zone, where broad expanses of snow offer little or no contrast for the photogrammetrist to work from. Considerable effort is devoted in the following sections to obtaining more generalized estimates of mapping error, and it is confirmed that actual errors are indeed significantly greater than those suggested by Blachut and Müller.
DIGITAL ELEVATION MODEL OF WHITE GLACIER

Preparation of the White Glacier DEM

The first DEM of terrain in the Expedition Fiord area was prepared in aid of an investigation of glacier surface albedo from satellite imagery (Cogley 1992; Jung-Rothenhäusler et al. 1992; Jung-Rothenhäusler 1993). This DEM (Figure 2) is an electronic version of the topography shown on the 1:10000 scale WG map (National Research Council 1965a), with accompanying landcodes describing land surface cover type.

Relying on contours and on printed spot heights and lake surface elevations, we read elevations on the map to the nearest metre by eye, using a transparent overlay. The overlay had printed on it a grid of crosshairs spaced every 5 mm in the horizontal and vertical directions over a range of 100 × 100 mm. Thus the grid was equivalent to an area of 1 km² on the ground and the interval between crosshairs translated to a spatial resolution of 50 m. A suitable frame was also drawn on the overlay to allow of registering it accurately to the grid lines of the map. Elevations, and changes of landcode, were read by one person either to a second person at a computer keyboard or to a tape recorder. Each square kilometre of readings was placed in a raw file which was then processed to separate the elevations from the landcodes and to generate a grid of the latter from the differential readings.

The Ermine River basin, in the southwest part of the White Glacier catchment, does not appear on the 1:10000 scale map. It was read with a similar overlay from the 1:50000 scale Thompson Glacier map.

The landcodes were single-byte codes standing for land, cliffs, ice and water. Cliffs are areas of steeply-sloping land labelled as “Rock wall” and mapped as blank, with no contours. Elevation estimates in such areas involve intelligent guesswork and are necessarily less accurate than where contours are available as guides. An additional code was used to represent unmapped portions of the rectangular DEM.

Measures for automated quality control included checks for more than 20 horizontal scanlines in a single file, correct ordering of the northings and eastings at the start of each scanline, more than 20 elevations in a single scanline, and the occurrence of local outliers with respect to their immediate neighbours. However systematic proofreading of the entire DEM by two persons was found to be essential in generating a correct product. One person read from a printout while the second verified the readout against the map. Coloured elevation and landcode maps, and a shaded-relief image, were also helpful in detecting errors. Separate 1 × 1 km files were kept for the raw and “fixed” stages (before and after quality control) of DEM generation.

Table 1 — Replicate Estimates of Elevation (White Glacier)

<table>
<thead>
<tr>
<th>Difference (m)</th>
<th>Raw measurements</th>
<th>After proofreading</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( f_1 - f_0 )</td>
<td>( g_1 - g_0 )</td>
</tr>
<tr>
<td>Minimum</td>
<td>-10.00</td>
<td>-9.00</td>
</tr>
<tr>
<td>Mean</td>
<td>0.09 ± 1.00</td>
<td>-0.10 ± 0.63</td>
</tr>
<tr>
<td>Rms</td>
<td>2.44</td>
<td>1.30</td>
</tr>
<tr>
<td>Maximum</td>
<td>25.00</td>
<td>8.00</td>
</tr>
</tbody>
</table>

| Minimum        | -10.00          | -9.00              | -10.00 |
| Mean           | 0.31 ± 0.69     | 0.28 ± 0.46        | 0.49 ± 0.97 |
| Rms            | 1.52            | 1.10               | 1.73   |
| Maximum        | 10.00           | 11.00              | 10.00  |

| Minimum        | -2.00           | -2.00              | -3.00 |
| Mean           | 0.40 ± 0.50     | 0.26 ± 0.38        | 0.37 ± 0.75 |
| Rms            | 0.77            | 0.68               | 0.94   |
| Maximum        | 3.00            | 4.00               | 3.00   |

Cell coordinates are the NRC coordinates of the southwest corner. Subscripts 0 and 1 refer to the first and second readings respectively. The differences \( f - g \) are, in each case, the greater in magnitude of \( f_0 - g_0 \) and \( f_1 - g_1 \). For means, error ranges are ± twice the average standard error \( \bar{\sigma} = (\sum_i \Delta_i^2/2)/N \), \( \Delta \) being the appropriate difference. In each 1 × 1 km cell the 50 m resolution of the DEM yields a sample size \( N = 400 \). Rms: root mean square, defined as \( \sqrt{((\sum_i \Delta_i^2)/N)} \).
Uncertainty in the White Glacier DEM

The data capture process included by design two replicate readings of 1 \times 1 \text{ km} cells, one in the accumulation zone and one in the ablation zone, by each of two readers (labelled “F” and “G”). The replicates were separated by several weeks, with one reading near the beginning of the work but after a period for learning, and the other near the end. Each reader worked alone. With each replicate consisting of a raw and an after-proofreading file, the resulting 2 \times 2 matrix made it possible to evaluate the reproducibility of each reader’s elevation estimates, the variability of estimates between different readers, and the effect of proofreading on the quality of the end product. These quantities are summarized in Table 1 and illustrated in Figures 3 and 4.

Figure 2. The digital elevation model of White Glacier. The shading interval is 100 m. The enclosing grey rectangle has dimensions of 16 \text{ km} \times 14 \text{ km}. Its southwest and northeast corners are at (23000,61000) and (39000,75000) respectively in NRC coordinates.
Figure 3. Frequency distributions of the difference in elevation estimates \( h \) made on the WG map by readers F and G in the 1 × 1 km cell with southwest corner at grid reference (29000,66000). The unshaded histogram represents differences before the proofreading stage. The shaded histogram represents differences after proofreading. Class width is 1 m, equal to the precision with which estimates were recorded.

Figure 4. Frequency distributions of the difference in elevation estimates \( h \) made on the WG map by readers F and G in the 1 × 1 km cell with southwest corner at grid reference (29000,66000). The shaded histogram represents observed differences before the proofreading stage (cf. unshaded histogram in Figure 3). The unshaded histogram represents the normal distribution having the same mean and variance as the shaded histogram.

The rows of Table 1 follow a pattern which is copied in several later tables. For each category (column) the minimum, mean, rms and maximum observed difference are given, as well as the number of samples (where it is not constant as in Table 1). The mean is accompanied by a formal error range based on the individual standard errors which, while it may or may not represent a 95% confidence interval, is nevertheless a robust indicator of the likelihood that the difference in question reveals a bias.

For example, all of the means in Table 1 are small with respect to their error estimates, and it is safe to conclude that the map-reading method described above yields unbiased estimates of elevation. There was no significant drift in the work of the two readers over time and no significant difference between their respective estimates. Mean differences were greater in magnitude in the accumulation-zone cell (29000,66000), perhaps because the slopes were gentler and therefore the contours were further apart. All but one of the mean differences was less in magnitude after proofreading than before. Likewise the minima and maxima grew smaller after proofreading, as a result of the elimination of gross errors.

The most useful of the statistics in Table 1 is probably the rms difference. In the absence of a source of “truth”, and of any reason to believe that one member of the difference is closer to the truth than the other, and of any evidence of bias, we can regard each estimate as a pair of random samples. The sampling process is repeated \( N \) times. If these repetitions are independent then the appropriate number to quote as a conventional estimate of error is the rms difference, and in what follows it will be our principal measure of uncertainty. The rms difference is proportional to the uncertainty in any single future sample of elevation, and granting certain assumptions this proportionality can be quantified. It is important, however, to be clear about what is being quantified: in the present context it is the uncertainty due solely to the reading of the map, and the rms differences in Table 1 contain no information about the fidelity of the map to the terrain.
In short, Table 1 suggests i) that the method adopted to create the WG DEM is unbiased; and ii) that map-reading errors are of the order of 1 − 3 m before proofreading and 0.7 − 1.3 m after. These numbers apply only to the 1:10 000 scale WG map, with its contour interval of 10 m.

Figure 3 shows a representative frequency distribution of between-reader elevation differences. The differences are tightly clustered about zero, especially after proofreading. The mean difference, 0.28 m from Table 1, is positive but appears unlikely to be significant. Figure 4 makes a point which, although it will not be repeated below, is important. It reproduces one of the histograms from Figure 3 along with an hypothetical histogram representing a normal distribution having the same mean and variance. The contrast between the two is striking, and demonstrates that the observed differences are not normally distributed. In general, all of the difference distributions in this study fail the standard tests for normality, such as the Kolmogorov-Smirnov test. This means that one of the usual assumptions made when quantifying error estimates is violated. The rms difference between any two related estimates of elevation or slope will still be proportional to the uncertainty, but we cannot be confident, for example, in the usual inference that there is a 95% probability that the true quantity lies within 2 rms differences of the estimated quantity. On the contrary, we can be confident that this probability will be somewhat less, or in other words that the suggested error bars are somewhat generous.

The most likely explanation for the non-normality of the observed differences is that they represent the sum of more than one population of errors. Several normally-distributed sources of random error, each of different variance, will add up to a distribution which in general will not be normally distributed. In addition, some kinds of error are themselves unlikely to be normally distributed.

![Figure 5. Row correlograms of the estimates of WG elevations by reader F in the 1 × 1 km cell with southwest corner at grid reference (24000,73000). The samples consist of pairs of differences f₁ − f₀ from rows (as opposed to columns) of the DEM. Dashed line: before proofreading; solid line: after proofreading. The dotted lines are at the magnitude of correlation r which differs from zero with probability 0.95 when sample size is 200 (the number of pairs of elevation estimates available at 50 m resolution and the maximum lag of 500 m).](image)

The other main assumption on which statistical estimates of uncertainty are usually based is that of independence of the samples. A spatial autocorrelation analysis, summarized in Table 2 and illustrated in Figure 5, was undertaken in the two replicate 1 × 1 km cells studied above. Each two-dimensional array of
Digital Elevation Models of Axel Heiberg Island Glaciers

elevation differences was correlated against copies of itself offset by varying distances (lags) in the easterly and southerly directions. All of the correlograms had the appearance seen in Figure 5. The decorrelation distances given in Table 2 are mostly less after proofreading, but they are still about 1–3 resolution elements. This suggests that adjacent elevation differences, and therefore presumably errors, are moderately correlated rather than independent. In short, we have found that mistakes exhibit a tendency to occur in clusters. This is another reason why we should conclude that the rms difference is a somewhat generous estimate of uncertainty in the DEM context.

Table 2 — Decorrelation of Elevation Estimates (White Glacier)

<table>
<thead>
<tr>
<th>Cell</th>
<th>Raw measurements</th>
<th>After proofreading</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rows Columns</td>
<td>Rows Columns</td>
</tr>
<tr>
<td></td>
<td>F G F G</td>
<td>F G F G</td>
</tr>
<tr>
<td>(29000,66000)</td>
<td>221 248 45 166</td>
<td>115 179 85 125</td>
</tr>
<tr>
<td>(24000,73000)</td>
<td>110 44 46 48</td>
<td>69 49 62 47</td>
</tr>
</tbody>
</table>

The quantities tabulated are the shortest lags (m) at which autocorrelation is not significantly different from zero at the 95% confidence level (upper dotted line in Figure 5). They were obtained by linear interpolation in correlograms such as that in Figure 5.

Summary

To summarize this investigation, errors in map-reading are unbiased but are not normally distributed and exhibit some mutual dependence. They are reduced markedly by careful quality control. The map-reading error in any single elevation estimate, about 1 m (Table 1), is a small fraction of the contour interval of 10 m.
**DIGITAL ELEVATION MODEL OF THE THOMPSON GLACIER REGION**

**Preparation of the Thompson Glacier DEM**

Most of the DEM of the Thompson Glacier region was prepared in essentially the same way as the WG DEM. The TG map has a contour interval of 25 m and is at smaller scale than the WG map, so the resolution chosen was 100 m rather than 50 m. Even at this coarser resolution the quantity of work was substantially greater than for the WG DEM. Minor improvements were made in the design of the grid overlay, and it was found convenient to introduce a coding sheet on which elevation estimates and landcodes were entered by hand before being transferred to computer files. The coding sheets reduced somewhat the incidence of mistakes and grew into a non-digital backup of the work. They also facilitated proofreading; nearly all of the map reading on the TG map was done by student assistants, while all of the formal proofreading was done by Reader G.

The nucleus of the TG DEM was a subsample of the WG DEM at 100 m resolution. As reading work progressed on the TG map, some of this subsample was replaced by TG estimates. This yielded a moderate number of replicate elevation estimates, one from each map, at coincident locations, allowing for between-map comparisons. The residual part of the TG DEM which is actually a low-resolution copy of the WG DEM appears on Figure 6 as the irregular area labelled “WG”.

The TG DEM was extended beyond the region covered by the TG map so as to include all of the glacier ice flowing towards Expedition Fiord. This led to the notion of the Expedition Fiord “glacier complex”, illustrated in Figure 6. It also required data collection from the EF map. This work, done by Reader G, was more difficult in a number of ways. The map itself is more generalized and more poorly controlled than the TG map. Crosshairs at 100 m resolution on the grid overlay were only 1 mm apart. The contour interval of 100 m, and the subdued relief of much of the upper accumulation zone of Thompson Glacier, called for considerable numbers of visual estimates over blank areas with no nearby contours. It was therefore decided to develop a new algorithm for interpolation of elevation estimates from digitized contours. This algorithm is explained and analyzed below. It was the source of most of the elevations in the part of the DEM to the north of northing 85000 m.

In its final state the TG DEM includes all of the Expedition Fiord glacier complex, all of the TG map, and some additional parts of the EF map.

The main findings of the error analysis of the WG DEM were accepted, and in further analysis attention was directed more towards between-map and between-method comparisons.

At an early stage in preparation of the TG DEM, glacier and lake outlines and drainage divides were digitized from the TG and EF maps. These are the basis of Figure 6, which portrays the DEM region independently of the DEM itself. The drainage divides were used to create a set of catchment codes with which the basic TG landcodes (cliff, land, ice and water) were augmented. The catchment codes are exploited in Table 3 to summarize the land-cover characteristics of the DEM region. Two thirds of the DEM proper is glacierized, water bodies occupy about 1%, and areas with inadequate contouring (cliff) occupy about 3%. Thompson Glacier is by far the largest glacier in the complex, especially if the confluent White Glacier and Wreck Glacier are added to it. Glacierization of the individual catchments ranges from negligible to more than 80%. The largest lake is Phantom Lake, in the east centre of the complex. Phantom Glacier and Transit Glacier have terminuses which calve into Phantom Lake, as does a distributary of Thompson Glacier. Astro Glacier and Finger Glacier also have calving terminuses.

The four basic landcodes (cliff, land, water, ice) were further supplemented with codes describing the ice cover types on the TG map: crevasses, moraine (“coarse morainic material” in the map legend) and fine debris (“fine debris and dust”). The distribution of these and of the basic landcodes is illustrated in Figure 7. Of the land portion of the DEM, 8.5% is coded as cliff. Of the ice portion, 8.2% is crevassed while 3.0% is mantled by moraine and 4.7% by fine debris.

The TG DEM itself is illustrated in Figure 8 and Figure 9.
Figure 6. The Expedition Fiord glacier complex, defined as the region draining to the head of Expedition Fiord, but including for simplicity parts of Hidden Ice Field which drain southwards to Strand Fiord. Glacier ice within the complex is shaded; lakes are black. Solid triangles: primary control points of the triangulation net (Haumann 1961, 1963); Astro 1 and Astro 2 are endpoints of the baseline and Level Point is the reference bench mark; one primary point, Upper Ice I at (36340,99686), lies beyond the north edge of the map. Open triangles: selected secondary control points. WG: area where elevation estimates in the TG DEM are samples every 100 m from the WG DEM; TG: area where elevations are measured from the TG map; EF: area where elevations are measured from the EF map. Grid labels (in km) are NRC coordinates.
Table 3 — Distribution of Landcodes by Catchment

<table>
<thead>
<tr>
<th>Catchment</th>
<th>Area (km²)</th>
<th>Ice</th>
<th>Water</th>
<th>Land</th>
<th>Cliff</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thompson Glacier</td>
<td>327.4</td>
<td>82.7</td>
<td>0.7</td>
<td>13.4</td>
<td>3.2</td>
</tr>
<tr>
<td>Astro Glacier</td>
<td>36.4</td>
<td>69.5</td>
<td>1.6</td>
<td>25.8</td>
<td>3.0</td>
</tr>
<tr>
<td>Bellevue Glacier</td>
<td>9.6</td>
<td>59.4</td>
<td>1.0</td>
<td>33.3</td>
<td>6.3</td>
</tr>
<tr>
<td>Finger Glacier</td>
<td>10.6</td>
<td>50.0</td>
<td>2.8</td>
<td>43.4</td>
<td>3.8</td>
</tr>
<tr>
<td>Hidden Ice Field</td>
<td>95.2</td>
<td>76.9</td>
<td>0.2</td>
<td>22.9</td>
<td>0.0</td>
</tr>
<tr>
<td>Little Phantom Gl</td>
<td>2.3</td>
<td>73.9</td>
<td>0.0</td>
<td>13.0</td>
<td>13.0</td>
</tr>
<tr>
<td>Phantom Glacier</td>
<td>30.1</td>
<td>38.5</td>
<td>17.6</td>
<td>27.6</td>
<td>16.3</td>
</tr>
<tr>
<td>Transit Glacier</td>
<td>59.1</td>
<td>83.1</td>
<td>0.2</td>
<td>14.0</td>
<td>2.7</td>
</tr>
<tr>
<td>White Glacier</td>
<td>70.1</td>
<td>62.1</td>
<td>0.3</td>
<td>35.0</td>
<td>2.7</td>
</tr>
<tr>
<td>Wreck Glacier</td>
<td>56.1</td>
<td>82.9</td>
<td>0.9</td>
<td>15.2</td>
<td>1.1</td>
</tr>
<tr>
<td>Crusoe Glacier</td>
<td>71.8</td>
<td>69.1</td>
<td>0.4</td>
<td>27.9</td>
<td>2.6</td>
</tr>
<tr>
<td>Wolf River Glacier</td>
<td>20.8</td>
<td>3.4</td>
<td>0.5</td>
<td>93.3</td>
<td>2.9</td>
</tr>
<tr>
<td>Expedition River</td>
<td>76.3</td>
<td>0.1</td>
<td>0.0</td>
<td>98.8</td>
<td>1.0</td>
</tr>
<tr>
<td>Total</td>
<td>865.8</td>
<td>67.3</td>
<td>1.2</td>
<td>28.6</td>
<td>2.9</td>
</tr>
<tr>
<td>Outside complex</td>
<td>586.1</td>
<td>26.7</td>
<td>2.2</td>
<td>16.2</td>
<td>54.9</td>
</tr>
</tbody>
</table>

See Figure 6 for catchment boundaries. “Cliff” means land (not ice) represented on the map by a cartographic cliff symbol, with contours reduced in number or absent; outside the glacier complex (last row) it also includes DEM cells with no recorded elevation.

Figure 7. Landcodes and icecodes of the Thompson Glacier DEM. Cliff: red; land: tan; water: light pink; ice: light blue. Crevasses: blue; moraine: brown; fine debris: cream. Crevasses were recorded for any ice cell with the linear crevasse symbol within 50 m of its centre; 40 cells (purple in the map) had both crevasses and morainic cover and 83 cells (orange) had both crevasses and fine debris.
Figure 8. The digital elevation model of Thompson Glacier and its neighbours. The shading interval is 100 m. The enclosing grey rectangle has dimensions of 45 km × 44 km. Its southwest and northeast corners are at (17700,51500) and (62700,95500) respectively.

**Errors in Map Reading**

Concerns about the accuracy of the map-reading method arise with the TG DEM just as with the WG DEM, and they are perhaps increased by knowledge of the smaller scale and coarser contour interval. It is also relevant that not all of the domain of the TG DEM is enclosed by the triangulation net which was the basis for mapping from air photographs.

Figure 10 and Table 4 compare the work of the four readers of the TG map who contributed to the TG DEM. A test rectangle with corners at (30000,69000) and (38000,70000) in NRC coordinates was the source of most of the replicate readings. It included a representative range of elevation, relief and land cover. The histograms, representing elevations after proofreading, are similar in appearance to that shown in Figures 3 and 4, but the horizontal scale is much broader in Figure 10. As Table 4 shows, most of the error measures are larger in magnitude than those obtained for the WG DEM. Mean differences, however, are all small with...
respective to their average standard errors, and there is therefore no evidence that the map readers are biased one with respect to another. Minimum and maximum differences have magnitudes in the dekametre range, and rms differences are $4 - 8$ m.

Figure 9. Shaded-relief image of the TG DEM. Glacier ice is blue and land is tan. The DEM is illuminated by an infinitely distant light source $25^\circ$ above the horizon in the northnortheast. Colour brightness is proportional to the cosine of the local angle of incidence of the light. The angle of incidence is computed from the DEM. It is the angle made by the outward normal of the terrain surface and a unit vector pointing to the light source.

The TG map has thus been read significantly less accurately than the WG DEM, but this is a relative conclusion. If we normalize by the contour interval $c$, the minimum and maximum differences have magnitudes of $0.2 - 0.8c$ (WG) and $0.5 - 2.5c$ (TG). The rms differences are about $0.1c$ (WG) and $0.2 - 0.4c$ (TG). These numbers are a little more favourable to the TG DEM than a comparison of Tables 1 and 4.
suggests. However it seems that gross errors were indeed harder to exclude from the TG DEM, perhaps because proofreading was less thorough. It was not possible to devote the same proportion of total effort to proofreading in the larger-scale TG project as in the WG project.

Table 4 introduces a second measure of DEM accuracy, the difference in calculated estimates of terrain slope. Given a sample of elevations, the number of slope estimates is typically smaller than the number of elevations because it is necessary to exclude the elevations of water bodies and slopes cannot be calculated at the edge of the sample. The finite-difference scheme used to estimate the slope at a point involves the elevations at its eight neighbours.

It would be possible to calculate the error in the slope given the errors in these elevations, but knowing that the latter errors are correlated and are not normally distributed it is safer to rely on empirical estimates based on differences in observed slopes. Mean differences are small and unbiased, there is no difference between the estimates from the different map readers, and the differences range up to 4° in magnitude. Rms differences are 0.4° – 0.9°. As for the elevations, caution is required in interpreting these statistics as confidence intervals because the usual statistical assumptions of independence and normality are not met. For slopes the situation is aggravated by the explicit dependence of adjacent estimates on each other: each slope estimate shares three contributing elevations out of eight with each of its neighbours.

<table>
<thead>
<tr>
<th>Difference</th>
<th>Elevation</th>
<th>Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reader M minus Reader C</td>
<td>Number</td>
<td>300</td>
</tr>
<tr>
<td>Minimum</td>
<td>−44.00</td>
<td>−1.33</td>
</tr>
<tr>
<td>Mean</td>
<td>0.26 ± 2.56</td>
<td>−0.04 ± 0.31</td>
</tr>
<tr>
<td>Rms</td>
<td>4.65</td>
<td>0.45</td>
</tr>
<tr>
<td>Maximum</td>
<td>26.00</td>
<td>2.35</td>
</tr>
<tr>
<td>Reader M minus Reader G</td>
<td>Number</td>
<td>400</td>
</tr>
<tr>
<td>Minimum</td>
<td>−62.00</td>
<td>−3.83</td>
</tr>
<tr>
<td>Mean</td>
<td>−1.89 ± 4.42</td>
<td>−0.02 ± 0.55</td>
</tr>
<tr>
<td>Rms</td>
<td>7.81</td>
<td>0.86</td>
</tr>
<tr>
<td>Maximum</td>
<td>14.00</td>
<td>3.99</td>
</tr>
<tr>
<td>Reader M minus Reader S</td>
<td>Number</td>
<td>699</td>
</tr>
<tr>
<td>Minimum</td>
<td>−28.00</td>
<td>−2.40</td>
</tr>
<tr>
<td>Mean</td>
<td>0.48 ± 4.32</td>
<td>−0.04 ± 0.44</td>
</tr>
<tr>
<td>Rms</td>
<td>6.44</td>
<td>0.64</td>
</tr>
<tr>
<td>Maximum</td>
<td>35.00</td>
<td>3.30</td>
</tr>
</tbody>
</table>

Elevation: Differences (m) between elevations measured by the different map readers M, C, G and S at coincident locations on the TG map. Slope: Differences (deg) in slopes computed from the work of the different readers.
Digital Elevation Models of Axel Heiberg Island Glaciers

Figure 10. Frequency distribution of differences in coincident elevation estimates from the TG map made by readers M, C, G and S. Reader M did most of the work and was chosen as the reference.

Errors in Mapping

Overlapping coverage of the various maps was built in to the data capture process so as to permit comparisons of elevation and slope estimates from the different sources. The WG map was drawn from aerial photographs taken from an altitude of 3050 m in August 1960. The TG map was drawn from August 1959 photographs taken from 9100 m. The EF map is based on the TG map over the extent of the latter, on August 1959 photography elsewhere in its southern portion, and on August 1960 photography in its northern portion. The WG and TG maps are therefore independent estimates of the state of the terrain. WG is one year later than TG, but the mass balance of White Glacier measured for that year (Cogley et al. 1996a) was the equivalent of −0.45 m of ice. As an elevation change, this is small compared to the differences to be discussed next. While the EF and TG maps are more closely related, the EF map is not simply a reduced-scale copy of the TG map, as will be seen.

Figure 11 and Table 5 summarize the distributions of differences in elevation and slope at locations common to the WG and TG DEMs. The magnitudes of the various difference statistics in the table are all larger than in previous comparisons, although evidence for the unbiased character of the DEM measurements remains as good as formerly. The rms differences, about 20 m in elevation and 2° in slope, point to substantially greater total uncertainty than in either DEM alone.

When comparing different maps, as opposed to comparing different readings of the same map, the members of any pair of coincident elevations need not have equal standing. Given comparably careful work the larger-scale map should be “better”, particularly if it has a smaller contour interval. On the other hand, neither map can be regarded as exact. With no knowledge of the true population of elevations, we are obliged to assume that the two samples are unbiased, but it would probably be a mistake to assign all of the random error to the smaller-scale map. For the WG and TG maps we have estimates of the map-reading uncertainty, based on Tables 1 and 4, but these are only part of the total error. The mapping error, which is the error in the map elevation as an estimate of the true elevation, remains unknown.
Figure 11. Frequency distribution of differences in coincident elevation (a) and slope (b) estimates from the TG and WG maps. In panel a the upper black portion of each bar represents the $N_b$ elevation differences from a locality where the two maps differ radically.

An error model which describes this situation is set out in Appendix A. To apply it to the comparison of the TG and WG maps, we will choose as estimates of the map-reading errors the median of the rms differences from Table 4, $r_T = 6.44$ m, and the median of the rms differences from Table 1, $r_W = 0.95$ m. From Table 5, elevation column a gives a comparison error of $\sigma_{TW} = 19.71$ m. Using (A5), the total mapping error is $m = 18.60$ m, so it is the dominant contributor to total error. Map reading has introduced only a small additional uncertainty. If we make the lumped-error assumption (Appendix A), and for the sake of illustration assign all of the error to the TG map, then from (A6) total error is $\gamma_T = 19.68$ m in the TG map and $\gamma_W = 0.95$ m in the WG map. The equable-error assumption of Appendix A gives $\gamma_T = 14.65$ m in the TG map and $\gamma_W = 13.19$ m in the WG map.

Table 5 — Between-map Differences (Thompson Glacier – White Glacier)

<table>
<thead>
<tr>
<th>Difference</th>
<th>Elevation$^a$</th>
<th>Elevation$^b$</th>
<th>Elevation$^c$</th>
<th>Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>2695</td>
<td>241</td>
<td>2454</td>
<td>1810</td>
</tr>
<tr>
<td>Minimum</td>
<td>-72.00</td>
<td>-43.00</td>
<td>-72.00</td>
<td>-8.50</td>
</tr>
<tr>
<td>Mean</td>
<td>5.56 ± 11.66</td>
<td>40.44 ± 43.34</td>
<td>2.13 ± 8.54</td>
<td>-0.02 ± 1.26</td>
</tr>
<tr>
<td>Rms</td>
<td>19.71</td>
<td>52.76</td>
<td>12.39</td>
<td>1.80</td>
</tr>
<tr>
<td>Maximum</td>
<td>99.00</td>
<td>99.00</td>
<td>66.00</td>
<td>8.01</td>
</tr>
</tbody>
</table>

Differences between measured elevations (m) and computed slopes (deg) at coincident locations on the TG and WG maps. a: all available elevations. b: elevations from a small area where the two maps differ radically; its southwest corner is at (30100,68900) and its northeast corner at (31300,71200). c: elevations excluding those in b.

These and related calculations are set out in the first row of Table 6. The reader is free to choose either, or neither, of the two assumptions about mapping error. There is nothing in this analysis to favour either of the maps. A conservative way to put these results to use in an investigation involving mapped elevations would be to adopt a figure of 20 m (lumped-error) or 15 m (equable-error) for the standard error of elevation.
Table 6 — Errors in Thompson Glacier and White Glacier Elevations

<table>
<thead>
<tr>
<th>Elevations</th>
<th>$\Gamma_{TW}$</th>
<th>$m$</th>
<th>Lumped</th>
<th>$\gamma_T$</th>
<th>$\gamma_W$</th>
<th>Equable</th>
<th>$\gamma_T$</th>
<th>$\gamma_W$</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>19.71</td>
<td>18.60</td>
<td>19.68</td>
<td>18.62</td>
<td>14.65</td>
<td>13.19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gross errors excluded</td>
<td>12.39</td>
<td>10.54</td>
<td>12.35</td>
<td>10.58</td>
<td>9.85</td>
<td>7.51</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In both rows $r_T = 6.44$ m and $r_W = 0.95$ m. The two rows correspond to elevation columns a and c of Table 5 respectively. See Appendix A and text for definitions of tabulated quantities; all are in metres.

A striking feature of the elevation distribution shown in Figure 11a is its positive tail. This tail comes almost entirely from a small area where the contours of the two maps differ radically. The area, on the east-central drainage divide of the catchment of White Glacier, appears as a ridge on the TG map. In effect, the TG representation of the divide is displaced westwards with respect to that of WG, and the TG map is up to 99 m higher. We do not have access to the air photographs from which the WG contours were drawn, but in the August 1959 TG photographs this area is a featureless white expanse, while contrast is much better in adjacent WG photographs from August 1960. It is therefore likely that the TG map is wrong.

Of more importance than assigning blame, however, is what this example suggests about the gross error rate. A point-by-point inspection of elevation differences over the area shows that the gross differences are organized into two sub-areas which are well delineated by the criterion $h_T - h_W > 35$ m; 129 of the differences are gross errors in this sense. This is 2.4% of the $2 \times 2695$ elevations available for estimating differences between TG and WG.

In both rows of Table 6 the mapping error $m$ is the largest contributor to total error $\gamma$. Thus the map-reading process, while it has added some uncertainty to the information from the map, is only moderately culpable.

For comparisons of the TG and EF maps (Figure 12 and Table 7) there are no replicate readings with which to estimate the map-reading error $r_E$. (Replicate measurements by different methods are discussed in a later subsection.) Map reading was more difficult on the EF map for reasons explained above, so $r_E$ is unlikely to be less than $r_T = 6.44$ m. But it is also likely that as in the TG-WG comparison the mapping error is the main contributor to total error: if $r_E = r_T$, say, we can use Appendix A to deduce that for the TG-EF comparison $m = 23.34$ m, accounting for most of the comparison error $\Gamma_{TE} = 25.05$ m (the rms difference from Table 7). Further, we obtain estimates for total EF-map error of $\gamma_E = 24.21$ m on the lumped-error assumption and $\gamma_E = 17.72$ m on the equable-error assumption.

On either assumption the total error is greater here than in the TG-WG comparison, tending to justify the inference that in between-map comparisons the greater part of the mapping error should be assigned to the map of smaller scale.

The histogram of elevation differences (Figure 12a) is noticeably skew. Consistent with the relatively large mean difference (Table 7), there is an excess of negative differences generally (EF higher than TG) and a negative tail which is reminiscent of the positive tail in Figure 11a. It was not possible to localize the negative tail, however, mainly because the replicate elevations are in narrow strips all around the northern and eastern edge of the TG map.
**Digital Elevation Models of Axel Heiberg Island Glaciers**

Figure 12. Frequency distribution of differences in coincident elevation (a) and slope (b) estimates from the TG and EF maps.

**Table 7 — Between-map Differences (Thompson Glacier – Expedition Fiord)**

<table>
<thead>
<tr>
<th>Difference</th>
<th>Elevation</th>
<th>Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>1059</td>
<td>518</td>
</tr>
<tr>
<td>Minimum</td>
<td>−92.00</td>
<td>−7.62</td>
</tr>
<tr>
<td>Mean</td>
<td>−11.58 ± 18.34</td>
<td>0.04 ± 1.69</td>
</tr>
<tr>
<td>Rms</td>
<td>25.05</td>
<td>2.46</td>
</tr>
<tr>
<td>Maximum</td>
<td>72.00</td>
<td>13.46</td>
</tr>
</tbody>
</table>

Differences between measured elevations (m) and computed slopes (deg) at coincident locations on the TG and EF maps. The work of all map readers was pooled for this comparison.

The estimated slope error grows more slowly between the TG-WG and TG-EF comparisons than does the elevation error: an increase by a quarter for slope against a doubling for elevation. This may be because of local autocorrelation of the elevation errors, as in Figure 5.

**Algorithm for Spatial Interpolation**

The northern part of the TG DEM, within which elevations were estimated by spatial interpolation, is shown in Figure 13. There are extra formlines at 1450 m and 1550 m (the former incomplete) in addition to the contours at multiples of 100 m. Most of the information north of northing 90000, and some to the south away from the centreline of Thompson Glacier, appears as dashed formlines (described as “indefinite” or “indeterminate” in the legend of the map). No account was taken of these distinctions. The topographic information was sparse in places where relief was particularly gentle.

Contours, formlines and spot heights on the EF map were digitized. Where topographic information was sparse, and where summits were without spot heights, the map was supplemented with spot heights estimated by eye (open triangles in Figure 13).

In the first or coarse stage of interpolation, each digitized contour was tagged with its elevation. The contours were sorted by elevation and ordered lists were made of all the distinct elevations and contour intervals (elevation bands). Each band was rasterized onto the DEM grid, each grid point being assigned the middle elevation of its band. (The bottom band was assigned an elevation less than the lowest contour elevation by one half of the lowest contour interval, with an analogous provision for the top band.) To accomplish this it was necessary to polygonize each elevation band in turn by assembling its bounding
Digital Elevation Models of Axel Heiberg Island Glaciers

contours and any necessary segments of the boundary of the interpolation region (Figure 13). For simplicity it was assumed (correctly) that none of the contours represented enclosed hollows. Grid points outside the band were filled with a “not done” code unless they already contained a recognizable elevation. The necessary special treatment was accorded to the top band. Once the sweep through the list of elevation bands was complete, the result was a coarsely quantized DEM containing only band-midpoint elevations.

Figure 13. Elevations in the northern part of the Expedition Fiord glacier complex. The catchment of Thompson Glacier is outlined with a heavy line and ice within it is shaded. Contours and formlines every 100 m (thin lines) and spot heights (solid triangles) were digitized from the EF map over the region illustrated. Within the rectangular area marked “Interpolation”, elevations were estimated by an interpolation algorithm described in the text, and the resulting grid is contoured at an interval of 100 m in the figure. (Contours are slightly thicker than the digitized contours; the extra digitized formline at 1550 m does not have an interpolated counterpart.) Open triangles are spot heights estimated from the map by eye and supplied as hints to the interpolation algorithm. Within the 3 × 5 km rectangular area marked “Comparison”, elevations were also estimated by eye as over the rest of the TG DEM. The visual estimates and those of the automated interpolation algorithm are compared in the text. Grid labels (in km) are NRC coordinates.

In the second, refinement stage, the band-midpoint elevation \( h_{mid} \) at each grid point was adjusted by adding a distance-weighted sum of nearby elevations. An adjustment set of \( I \) known elevations \( h_i \) at distances \( d_i \) from the grid point was assembled, consisting of the nearest points on the lower and upper bounding contours and any sufficiently close spot heights within the elevation band. This scheme resulted almost always in \( I \geq 2 \), and at 94% of grid points \( I \) was either 2 or 3 (Table 8). Several of the distance-decay functions commonly used in spatial interpolation were explored before a simple square-root rule was selected. That is, the weight of the \( i \)th elevation in the adjustment set was computed as

\[
w_i = \frac{1}{\sqrt{d_i}}
\]
and used in the expression

\[ h_{intp} = h_{mid} + \sum_{i=1}^{I} \frac{w_i (h_i - h_{mid})}{\sum_{i=1}^{I} w_i} \]

to calculate the refined elevation \( h_{intp} \) at the grid point.

The interpolated contours in Figure 13 show that this algorithm yields very satisfactory results. See Almansa et al. (2002) for a recent discussion of algorithms for spatial interpolation. The present algorithm is free of the complications (divergence, overshoots) which can afflict polynomial-based interpolation algorithms (such as splines under tension; Wahba 1990). One of its weaknesses is that in peculiar situations it may not cope well with local extrema (hilltops, hollows) which are not global extrema. For example, the adjustment set of a local summit will contain its lower bounding contour, as it should, but if there happens to be an instance of the upper bounding contour within the maximum distance then it too will be placed, wrongly, in the adjustment set.

Table 8 — Distribution of Number of Samples used for Interpolation

<table>
<thead>
<tr>
<th>Samples used</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5-100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instances</td>
<td>0</td>
<td>1</td>
<td>9124</td>
<td>4953</td>
<td>904</td>
<td>18</td>
</tr>
</tbody>
</table>

Samples used: Number of contour and spot elevations used in refinement of coarsely interpolated elevations. Instances: At 100 m resolution the 15 × 10 km interpolation region contains 15 000 DEM points.

The distance beyond which candidate elevations were excluded from adjustment sets was chosen as \( d_{max} = 2205 \) m. By this distance the weight \( w \) has decreased to about 0.02.

Errors of Interpolation

To gauge the comparability of elevations estimated by the interpolation algorithm and by the visual method, visual measurements were made over a comparison region (Figure 13). The three resulting contour sets are illustrated in Figure 14, where it is seen that, as expected, agreement between the digitized EF map, the interpolated DEM and the visually-generated DEM is very close but not perfect.

The distribution of elevation differences is graphed in Figure 15 for the two stages of interpolation. The difference between visual estimates and coarse interpolates appears to obey a uniform distribution over a range of approximately −50 m to +50 m, which is equal in magnitude to the contour interval. The differences are unbiased (Table 9) and the rms difference is consistent with the hypothesis of a uniform distribution. None of this is unexpected.

The difference between visual estimates and refined interpolates is less dispersed. The interpolation algorithm appears to have had a moderate tendency to drag elevations upwards, the interpolates being 4 m higher on average, but this is not of statistical significance. The rms difference is much reduced from the coarse stage. When normalized by contour interval \( c = 100 \) m, this between-method difference of 0.13 is comparable to the between-reader differences obtained on the WG map (0.07 – 0.13, Table 1) and smaller than those obtained on the TG map (0.18 – 0.31, Table 4). Of course it is not strictly comparable to those estimates of map-reading error, but it is the only guidance available and we therefore adopt it as an estimate of the map-reading error for the EF map, \( r_E = 13.05 \) m.
Figure 14. Elevations in comparison region (Figure 13). Thin lines: contours digitized from EF map. Thick solid lines: contours of elevations estimated by interpolation algorithm. Thick dashed lines: contours of elevations measured by eye from map. a: contour interval 100 m, the lowest contour being at 900 m (southeast corner). Extra formline at 1550 m (west centre) has no interpolated or measured counterpart. b: as in panel a, but dashed contours of measured elevations are at 950, 1050, . . . m.

Equipped with this new estimate of $r_E$, we can revisit the analysis of Table 7, in which we were seeking to separate mapping and map-reading errors on the TG and EF maps and to estimate total errors for each map. Recall that the comparison error in the TG-EF comparison was $\Gamma_{TE} = 25.05$ m, and that $r_T$ was estimated as 6.44 m. This leads to a new estimate from (A5) of total mapping error $m = 20.42$ m. In turn this gives us a total error for the EF map of $\gamma_E = 24.21$ m if all of the mapping error is lumped onto the EF map, or $\gamma_E = 19.43$ m if the mapping error is equably distributed between the two maps.

Table 9 — Coarse and Refined Elevation Differences (Expedition Fiord)

<table>
<thead>
<tr>
<th>Difference</th>
<th>Coarse</th>
<th>Refined</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>1500</td>
<td>1500</td>
</tr>
<tr>
<td>Minimum</td>
<td>-80.00</td>
<td>-78.86</td>
</tr>
<tr>
<td>Mean</td>
<td>0.28 ± 61.80</td>
<td>-4.27 ± 24.66</td>
</tr>
<tr>
<td>Rms</td>
<td>30.89</td>
<td>13.05</td>
</tr>
<tr>
<td>Maximum</td>
<td>81.00</td>
<td>45.90</td>
</tr>
</tbody>
</table>

Differences (m) are between coincident measured and interpolated elevation estimates from the comparison region of the EF map. Their distribution is shown in Figure 15.
Summary

Map-reading errors are significantly greater in the TG DEM (∼ 4 – 8 m) than in the WG DEM (∼ 1 m), and appear in turn to be greater still in the smaller-scale EF portion of the TG DEM (∼ 13 m). This trend is understandable in terms mainly of decreasing scale and increasing contour interval, although less thorough proofreading of the TG DEM also plays a part. The interpolation algorithm developed for the northern EF portion performed well by comparison with the visual measurements made over the larger part of the TG DEM.

Although a rigorous separation is not possible, total error in estimated elevations is due largely to errors in the maps and only secondarily to errors in map reading. The total error lies between 1 and 19 m for the WG DEM; between 6 and 20 m for the 1:50 000 part of the TG DEM; and between 13 and 24 m for the 1:100 000 (EF) part of the TG DEM. Blunders, probably at the photogrammetric stage, have a major impact on uncertainty. The blunder rate seems to be rather high. The evidence for this is slight in bulk, but if valid the conclusion is likely to hold beyond the maps studied here.

Errors in calculated slopes grow more slowly with the contour interval than do elevation errors, but it is harder to quantify confidence intervals for slope errors.
COMPARISONS WITH SMALL-SCALE DIGITAL ELEVATION MODELS

Small-scale Digital Elevation Models

The WG and TG maps and DEMs are distinctive in being based on local ground control and having detailed, if incomplete, supporting documentation. There are, however, other sources of digital topographic information about the Expedition Fiord area.

Among digital products, those of most relevance here are the vector files published as part of NTDB, the National Topographic Data Base (Centre for Topographic Information 1999) and, more directly, the digital elevation model CDED (Centre for Topographic Information 2000). These are digital renditions of the NTS 1:250 000 scale maps. NTDB contains contours and other vector information. CDED, a raster product, is interpolated from NTDB information using the ANUDEM software (Hutchinson 1989, 1996). Both products are referenced to the NAD83 horizontal datum and to Mean Sea Level as defined in the Canadian Vertical Geodetic Datum.

The CDED DEM, presented in geographical coordinates, has meridional resolution of 3″ and zonal resolution of 6″, or 92 m by 34 m. For the Expedition Fiord area, the parent map is the NTS Strand Fiord map sheet 59H, which has a contour interval of 152 m (500 feet). The aerial photographs which are the source of the NTS map information are the same as those used in preparation of the TG map.

For the purpose of comparison of CDED with the TG DEM, TG grid point positions were recast into WGS84 and their CDED elevations assigned from the nearest CDED elevation (Figure 16c). Another resampling algorithm, cubic convolution (e.g., Billingsley 1983), was also used as an alternative and appears in some of the comparisons below. It was not thought profitable to investigate possible differences between CDED’s vertical datum and the local vertical datum of the TG DEM.

Landcodes (representing land, ice and water) for the CDED DEM of the Expedition Fiord area were generated from NTDB. The NTDB vector information can be regarded in the present context as having a spatial resolution of 2.5″. It was not obtained directly from NTDB vector files but rather from a GIF (Graphics Interchange Format) file on Toporama, the Natural Resources Canada web site for NTDB (http://toporama.cits.rncan.gc.ca). The resolution of the GIF file, 6400 × 1600 pixels for a map domain of 4° × 1°, translates to a cell size of 2.5″, or about 77 × 14 m at the latitude of Expedition Fiord.

The GIF file was cleaned so as to eliminate all information other than that relating to land cover, that is, to leave only codes representing land, ice, water or a boundary between one of these cover types. Boundary pixels were replaced with the most numerous landcode among neighbouring landcode pixels, ties being resolved in favour of land (or water for ties between water and ice). Nearest-neighbour resampling of this landcode model yielded a landcode model identical in format with that of the TG DEM.

The accuracy of CDED and NTDB is not specified in detail. It is stated that NTDB “aims at attaining” planimetric (positional) accuracy of 125 m (Centre for Topographic Information 1999), while “in some NTDB data sets, horizontal inaccuracy goes up to . . . 500 metres” (Centre for Topographic Information 2000). No numerical estimates are provided for the accuracy of CDED elevations.

At a scale still smaller than that of CDED, the Digital Chart of the World (DCW; Defense Mapping Agency 1992) and the related GLOBE DEM (Hastings et al. 1999) cover Axel Heiberg Island at potentially useful resolution. At the latitude of Axel Heiberg Island the 30″ resolution of GLOBE corresponds to meridional and zonal resolutions of 930 m and 170 m respectively. For the Canadian High Arctic, the GLOBE elevation data derive from Operational Navigation Charts, a series of aeronautical charts with worldwide coverage produced by the U.S. Army Mapping Service (now NIMA, the National Imagery and Mapping Agency). They are 1:1 000 000 scale maps based on Corona satellite imagery of the early 1960s with typical scales of 1:300 000 to 1:400 000. The contour interval is 305 m (1000 feet).

Like CDED, GLOBE is interpolated from vector information using ANUDEM, in this case the vector source being DCW. GLOBE is referenced to the WGS84 horizontal datum and to Mean Sea Level. “Absolute horizontal accuracy of the DCW hypsography is reported to be 2040 m rounded to the nearest 5 m at 90%
circular error. Vertical accuracy is considered to be 610 m for contours, and 30 m for spot elevations” (Hastings et al. 1999).

GLOBE elevations were resampled to the TG DEM grid by nearest-neighbour (Figure 16a) and cubic-convolution resampling, as for the CDED elevations. Landcodes for the GLOBE DEM of the Expedition Fiord area were taken unchanged from the CDED DEM.

Figure 16. Different DEMs of the Expedition Fiord area. All are on the transverse Mercator grid of NRC coordinates. a: The GLOBE DEM, derived by nearest-neighbour resampling of the 30” GLOBE dataset. b: The TG DEM, as shown in Figure 8. c: The CDED DEM, derived by nearest-neighbour resampling of the 3” × 6” CDED dataset for NTS map sheet 059H. d: TG elevation minus CDED elevation (panel b minus panel c).

The TG DEM as shown in Figure 16b may be compared directly with GLOBE (Figure 16a) and CDED (Figure 16c). Both GLOBE and CDED are visibly more generalized than the TG DEM, an obvious consequence of their derivation from maps of smaller scale. Although the native resolution of CDED is comparable with that of TG, the DEM contains only information which is present or implied in its parent 1:250 000 scale map. The parent map of GLOBE is at 1:1 000 000 scale and is yet more generalized. The
Digital Elevation Models of Axel Heiberg Island Glaciers

quantization of GLOBE is also striking; the low resolution of the parent DEM means that each elevation in GLOBE appears roughly 2 × 9 times at the 100 m resolution of the TG DEM.

To illustrate the differences which are analyzed in more detail in the next section, Figure 16d shows the quantity \( h_T - h_C \). It demonstrates that differences can be large, exceeding 100 m in magnitude over substantial parts of the DEM domain. CDED is much higher than TG in the northwest and significantly lower in the centre and northeast. CDED is unable to resolve the Expedition River sandur in the southwest, where it is significantly too high because it lacks a source of information below the lowest contour at 150 m. (One of the strengths of ANUDEM is its exploitation of information on watercourses to enhance the realism of interpolation, and Figure 16c suggests that it has indeed done a good but in this instance mistaken job in this respect.) Where the differences are smaller, as in the southeast and east, there is evidence of rich high-frequency detail (or noise) in the difference field.

Comparisons

We compare GLOBE and CDED to TG in a manner analogous to that used for between-map comparisons in the previous section. In this section, however, each of the three DEMs is complete and sample sizes are therefore much larger, justifying some changes in mode of presentation.

Figure 17 illustrates the comparison between CDED and TG. Figure 17a shows the hypsometric frequency distribution of the two DEMs. There is a clear resemblance, but some of the discrepancies are also striking. CDED reaches higher elevations than does TG and has more terrain at the highest elevations, above 1600 m, and less at 1400 – 1600 m. It has significantly more terrain at 50 – 200 m and much less at 0 – 50 m. Its histogram is noticeably more jagged than that of TG, exhibiting a repeating three-bar pattern which suggests a connection with the contour interval, \( \sim 150 \) m.

Figure 17b is the equivalent of Figure 17a for the frequency distribution of slopes. The two DEMs have comparable frequencies of slopes less than 1°. The modal interval is 1-2° for both of them, but CDED has a relative excess of slopes of 1-5° while TG has a relative excess of steeper slopes. This means that the terrain of CDED is smoother and less rugged than that of TG.

The histogram of elevation differences in Figure 17c is extremely broad by comparison with those of the previous section. The peaks at the left and right of the graph arise simply because the distribution is truncated. The histogram of slope differences in Figure 17d is more regular. There is a slight but noticeable skewness to the right (TG steeper than CDED), which is consistent with the relative excess of steeper slopes noted for TG in Figure 17b. Neither in elevation differences nor in slope differences, however, is there a significant bias away from an expected value of zero.

These assertions are justified quantitatively by reference to Tables 10 and 11, which summarize the difference distributions in the format used in previous sections. Figure 17 corresponds to the leftmost numerical column of each table. Elevation differences between TG and CDED reach several hundred metres, and although the two DEMs are evidently unbiased with respect to one another the rms difference between them approaches 100 m. The greatest differences in slope are almost as large as the steepest observed slopes \( (35-36° \text{ for TG and 32-33° for CDED}) \), and the rms difference in slope is not much less than \( 5° \).

Figure 18 summarizes the comparison of the TG and GLOBE DEMs. The summary might itself be summarized by saying that GLOBE is like CDED but more so. There is a similar excess of the highest elevations in GLOBE relative to TG, and a similar relative deficit of the lowest elevations (Figure 18a). The jaggedness of its hypsometric distribution, with modes centred at multiples of its contour interval \( c = 1000' \), is more marked. (GLOBE has a supplementary contour at 500'.) GLOBE has a large relative excess of gentle slopes (Figure 18b); the modal interval, 0-1°, contains 30.1% of all the slope samples, but in large part this is an artefact of oversampling (the tiling seen in Figure 16a). TG has a relative excess of slopes in the range 2-26°, but beyond 26° GLOBE has an excess. This is all but invisible in Figure 18b because the number of points involved is tiny, but the GLOBE histogram extends beyond the steepest slopes in TG (at 36°) to include a few which exceed 50°.
Figure 17. Distributions of elevation and slope in the CDED DEM. Elevations were transferred to the transverse Mercator grid of the TG DEM by nearest-neighbour resampling. Sample size is of the order of 110,000 in each panel. a: Elevation (shaded histogram), with the hypsometry of the TG DEM overlaid as the unshaded histogram; class interval 50 m. b: Slope (shaded histogram), with the slope distribution of the TG DEM overlaid as the unshaded histogram; class interval 1°. c: TG elevation minus CDED elevation; class interval 10 m. d: TG slope minus CDED slope; class interval 0.5°.

Table 10 — Between-DEM Elevation Differences

<table>
<thead>
<tr>
<th>Difference</th>
<th>Nearest neighbour</th>
<th>Cubic convolution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TG-CDED</td>
<td>TG-GLOBE</td>
</tr>
<tr>
<td>Minimum</td>
<td>−412.00</td>
<td>−618.00</td>
</tr>
<tr>
<td>Mean</td>
<td>3.49 ± 71.74</td>
<td>−30.45 ± 122.96</td>
</tr>
<tr>
<td>Rms</td>
<td>92.45</td>
<td>159.62</td>
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<tr>
<td>Maximum</td>
<td>377.00</td>
<td>682.00</td>
</tr>
</tbody>
</table>

Differences (m) are between coincident elevation estimates from the three DEMs. See also Figures 17, 18.
Table 11 — Between-DEM Slope Differences

<table>
<thead>
<tr>
<th>Difference</th>
<th>Nearest neighbour</th>
<th>Cubic convolution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TG-CDED</td>
<td>TG-GLOBE</td>
</tr>
<tr>
<td>Minimum</td>
<td>−25.44</td>
<td>−38.45</td>
</tr>
<tr>
<td>Mean</td>
<td>0.73 ± 3.05</td>
<td>2.01 ± 4.99</td>
</tr>
<tr>
<td>Rms</td>
<td>4.44</td>
<td>6.95</td>
</tr>
<tr>
<td>Maximum</td>
<td>28.28</td>
<td>32.80</td>
</tr>
</tbody>
</table>

Differences (deg) are between slopes calculated from coincident elevation estimates from the three DEMs. See also Figures 17, 18.

Figure 18. Distributions of elevation and slope in the GLOBE DEM. Elevations were transferred to the transverse Mercator grid of the TG DEM by nearest-neighbour resampling. Sample size is of the order of 110,000 in each panel. a: Elevation (shaded histogram), with the hypsometry of the TG DEM overlaid as the unshaded histogram; class interval 50 m. b: Slope (shaded histogram), with the slope distribution of the TG DEM overlaid as the unshaded histogram; class interval 1°. c: TG elevation minus GLOBE elevation; class interval 10 m. d: TG slope minus GLOBE slope; class interval 0.5°.

The distributions of TG-GLOBE elevation and slope differences (Figure 18c,d) are still more dispersed than those for TG-CDED. Elevation differences (Table 10, second column) exceed 600 m in magnitude and the rms difference is well over 100 m. Slope differences (Table 11, second column) exceed 30°, with an rms difference of 7°. Consistent with the appearance of the slope distributions in Figure 18b, the difference histogram (Figure 18d) is strongly skewed to the right, TG being a markedly steeper DEM than GLOBE. The mode of Figure 18d is at 0.5-1.0° but the mean difference is 2°. Because of the dispersion of the sample
Digital Elevation Models of Axel Heiberg Island Glaciers

as a whole it is not possible to ascribe statistical significance to this mean as differing from zero, but still it is substantial as a fraction of the average standard error.

The CDED and GLOBE DEMs which were obtained by cubic-convolution resampling are not shown, but they appear in the form of summary statistics in Tables 10 and 11. Comparison of the first numerical column of each table with the third shows that for CDED the cubic-convolution DEM differs only slightly from the nearest-neighbour DEM. A map and a set of histograms confirm that departures from the nearest-neighbour DEM are subtle. Most of the small differences are in favour of cubic convolution, but they scarcely repay the added complexity. Cubic convolution interpolates an estimate of the resampled variable based on the fitting of cubic polynomials in a $4 \times 4$ grid surrounding the desired location. As suggested by the fourth column in Table 10 and Table 11, the situation for the GLOBE cubic-convolution DEM is different. It contains abundant kilometre-scale elevation artefacts, has an rms elevation difference of almost 200 m, and features slopes differing from their TG equivalents by up to $70^\circ$.

On the basis of these results it was concluded that nearest-neighbour resampling was appropriate as a tool for mapping the smaller-scale DEMs into the domain of the TG DEM. The cubic-convolution algorithm had the potential for exploiting spatial autocorrelation among neighbouring elevations to produce a more realistic interpolated field. However it showed to little advantage with CDED and was defeated by the low resolution of GLOBE, which required the use of information from as far as 2600 m away from the interpolation point. Although it is not a formal requirement of cubic convolution that grid spacing be uniform in the two coordinate directions, the great difference between GLOBE's zonal and meridional resolutions may also have degraded performance. Finally, beyond these technical shortcomings there is the fact that, relative to TG and even to CDED, GLOBE is a highly generalized representation of the terrain.

Of the types of error defined in Appendix A, the only one we can identify unambiguously for CDED or GLOBE is the comparison error $\Gamma_{TC}$ or $\Gamma_{TG}$, which from (A4) is a function of the mapping and map-reading errors of TG and either CDED or GLOBE. The TG errors were discussed in detail earlier, but here “TG” is a composite of TG proper (from the 1:50 000 scale TG map) and EF (from the 1:100 000 scale EF map). To allow for this mixture, and for the purpose of discussion, we will adopt a total error $\gamma_T = 20$ m for the TG DEM (see the summary of the last section). Taking the comparison errors from the rms differences in the first and second columns of Table 10, and using (A4) and (A6), we obtain the following estimates of the total errors in the CDED and GLOBE DEMs:

$$\gamma_C = \sqrt{\Gamma_{TC}^2 - \gamma_T^2} = 90.26 \text{ m}; \quad \gamma_G = \sqrt{\Gamma_{TG}^2 - \gamma_T^2} = 158.36 \text{ m}.$$  

Thus, because $\gamma_T$ is so much smaller than $\Gamma_{TC}$ and $\Gamma_{TG}$, we might as well have assumed it to be zero. The 2-m difference between $\gamma_C$ and $\Gamma_{TC}$, for instance, is unlikely to be of consequence in any working context where error estimates are needed for elevations.

We do not have the means of separating total error into mapping error and map-reading error for either of the small-scale DEMs, but it seems likely that as for the Expedition Fiord DEMs the mapping error will be larger than the map-reading error. In the next section, however, we discuss estimates of one of the components of map-reading error obtained by a different approach than that used hitherto.

**Spurious Low-digit Noise**

The possible contour-interval artefacts in Figures 17a and 18a warrant further investigation, which we conducted in terms of the quantity $\eta = h \mod c$, the remainder (in metres) when the elevation $h$ is divided by the contour interval $c$. $\eta$ varies from 0 to $c$ as one travels upwards across any one elevation band between neighbouring contours. For the purpose of this investigation five DEMs are available: WG, TG with its TG-proper and EF portions considered separately, CDED and GLOBE. Figure 19 shows the observed frequency distributions $f(\eta)$ for each of the five DEMs.

Each DEM has a different contour interval, and none of the $c$ is close to any known physiographic or glaciological length scale. The natural hypothesis is thus that $\eta$ should be a random variable uniformly
distributed between 0 and c, that is, \( f(\eta) = \text{const} \). We consider observed departures from uniformity in terms of the following expression:

\[
f(\eta) = f_{\text{unif}} \pm \phi_{\text{phys}} \pm \phi_{\text{samp}} \pm \phi_{\text{algo}}
\]

(3)

where \( f_{\text{unif}} = 1/c \text{ m}^{-1} \) is the constant frequency expected for a uniform distribution. The \( \phi \) are random variables. \( \phi_{\text{phys}} \) represents “real” or physiographic departures from uniformity. Such departures are possible, and perhaps likely, within any one elevation band, but we have no means of identifying them and moreover they should be expected to cancel rapidly as the number of bands \( (h_{\text{max}} - h_{\text{min}})/c \) increases. In Figure 19, panel e (GLOBE) represents seven superimposed bands and the others represent more. We therefore assume that \( \phi_{\text{phys}} = 0 \), and that the departures from uniformity which can be seen in the panels of Figure 19 represent sampling variability, \( \phi_{\text{samp}} \), and shortcomings in methodology, \( \phi_{\text{algo}} \).

Figure 19. Distributions of \( \eta = h \mod c \), the remainder of DEM elevation divided by source contour interval \( c \). The histogram expected on the hypothesis that \( \eta \) is uniformly distributed is shown as a horizontal line. a: WG DEM; \( c = 10 \text{ m} \). Scale on frequency axis differs from that of other panels. b: TG DEM, including only parts obtained from the 1:50000 scale TG map with \( c = 25 \text{ m} \). c: TG DEM, including only parts obtained from the 1:100000 scale EF map with \( c = 100 \text{ m} \). d: CDED DEM (nearest-neighbour resampling); \( c = 500' \). e: GLOBE DEM (nearest-neighbour resampling); \( c = 1000' \).

It is not easy to separate these two contributions, but we suggest that sampling variability explains the high-frequency variation in the graphs (the “wiggles”) while methodological problems account for the broader structures. Crude estimates of the standard deviation of \( \phi_{\text{samp}} \) based on this interpretation are offered in Table 12. If they are correct then, except possibly for WG, the methodological problems are responsible for most of the departures from uniformity.

Table 12 — Attributes of the DEMs

<table>
<thead>
<tr>
<th>Quantity</th>
<th>WG</th>
<th>TG</th>
<th>EF</th>
<th>CDED</th>
<th>GLOBE</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c )</td>
<td>10.0</td>
<td>25.0</td>
<td>100.0</td>
<td>152.4</td>
<td>304.8</td>
</tr>
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<td>( N )</td>
<td>21710</td>
<td>85596</td>
<td>21894</td>
<td>111526</td>
<td>111526</td>
</tr>
<tr>
<td>( f_{\text{unif}} )</td>
<td>0.10</td>
<td>0.04</td>
<td>0.01</td>
<td>0.007</td>
<td>0.003</td>
</tr>
<tr>
<td>( (h_{\text{max}} - h_{\text{min}})/c )</td>
<td>178</td>
<td>74</td>
<td>10</td>
<td>14</td>
<td>7</td>
</tr>
<tr>
<td>( r_{\text{samp}} )</td>
<td>( 10^{-2} )</td>
<td>( 10^{-5} )</td>
<td>( 10^{-4} )</td>
<td>( 10^{-4} )</td>
<td>( 10^{-4} )</td>
</tr>
<tr>
<td>( r_{\text{algo}} )</td>
<td>0.28</td>
<td>0.29</td>
<td>0.61</td>
<td>3.76</td>
<td>16.01</td>
</tr>
</tbody>
</table>

\( c \): contour interval (m). \( N \): sample size. \( f_{\text{unif}} = 1/c \text{ m}^{-1} \): frequency expected for \( f(\eta) \) on the hypothesis that \( \eta \) is uniformly distributed between 0 and \( c \). \( (h_{\text{max}} - h_{\text{min}})/c \): number of contour-bounded elevation bands contributing to estimates of \( f(\eta) \). \( r_{\text{samp}} \) (m): standard deviation of sampling variability \( \phi_{\text{samp}} \) (see text), estimated by eye from Figure 19. \( r_{\text{algo}} \) (m): map-reading error due to lack of methodological realism, estimated using equation (4).
Each distribution is distinctive and peculiar. For WG (panel a), the units digit 0 occurs appreciably more often than the other nine possibilities, while for TG-proper (panel b) there are too few occurrences of \( \eta \in \{8 \ldots 17\} \) and too many of \( \eta \in \{0 \ldots 7\} \) and \( \eta \in \{18 \ldots 22\} \). These results have emerged in spite of all the map readers having schooled themselves explicitly not to favour any particular low-order digit or digits, a prescription based on awareness of the problem from earlier work (Briggs 1989). The four map readers who contributed in different proportions to panel b have quite different distributions of \( \eta \) (not shown). Each reader has evidently adopted a different approach to the psychovisual problem of interpolation between contours. It appears in particular that the sharp changes near \( c/3 \) and \( 2c/3 \) in Figure 19b are accidental rather than evidence of a flaw common to all of the readers.

For EF, in which elevations are interpolated rather than measured by eye, panel c suggests that, sampling variations apart, the interpolation algorithm (2) has a moderate tendency to avoid elevations just above contours (\( \eta \) near to zero), and perhaps a tendency to favour elevations somewhat below the mid-point elevation \( h_{\text{mid}} (\eta = c/2) \) at the expense of elevations somewhat above. Neither of these tendencies, however, is explicit in equation 2, and qualitatively one might judge panel c to be the “best” of the five.

Both CDED (panel d) and GLOBE (panel e) have too many elevations near to contours and too few in between; that is, they have spurious gently-sloping shelves with central elevations an integer multiple of \( c \), separated by spurious steeper banks in the middle of contour elevation bands. The distributions plotted here are those of the resampled DEMs covering the TG domain, but we have verified that the parent DEMs, both in geographical coordinates, have equivalent distributions of \( \eta \). For GLOBE, the multiple spikes seen at both ends of the range are not understood. Nor is the more subdued pattern seen at the low end of CDED’s range. The shared basic shape may be traceable to the interpolation algorithm of ANUDEM, which is also shared by the two DEMs, but any of the algorithmic artefacts could have appeared at other undocumented stages of DEM preparation.

Figure 19 offers a different angle from which to attack the question of uncertainty in DEM elevations. If we accept the hypothesis that \( \eta \) is in truth uniformly distributed, it follows that elevations must be in error by an amount sufficient to restore the observed distribution to uniformity. Exact uniformity, \( f(\eta) = f_{\text{unif}} \), is not entirely realistic, but it is an objectively defined state which is reachable readily by reassigning the observed positive and negative departures. We did this for each of the DEMs, minimizing the magnitude of the difference \( \Delta \eta \) for each reassignment. Each negative departure from \( f_{\text{unif}} \) was eliminated by drawing upon the nearest positive departure to its left (lower \( \eta \)). This led us to a quantity

\[
 r_{\text{algo}} = \frac{\sum \eta n \Delta \eta}{N} \tag{4}
\]

where \( n \) is the number of elevations reassigned by the amount \( \Delta \eta \) and \( N \) is total sample size. \( r_{\text{algo}} \), the root of the variance of \( \phi_{\text{algo}} \), may be interpreted as an algorithmic error in elevation which contributes to the map-reading error \( r \) discussed earlier. (Note that equation 4 assumes that \( r_{\text{samp}} \) is zero.) Table 12 shows that in general \( r_{\text{algo}} \) makes a moderate contribution. For WG it is equal to about 30% of the map-reading error, but this may be an overestimate because sampling variability is quite large in this instance. For TG and EF, \( r_{\text{algo}} \) is less than 5% of \( r \). The map-reading error is not known for CDED and GLOBE, but \( r_{\text{algo}} \) is small by comparison with total error. Across the five DEMs it is equal to between 1 and 5% of the contour interval. Thus, while at first sight the departures from randomness in Figure 19 are disconcerting, in practice they are not the principal reason for being uncertain about DEM elevations.

**Frequency Distributions of Slope**

In Table 5 we compared slopes calculated on the WG and TG maps, obtaining an rms difference of 1.8°. The contributing elevations were measured on different maps at coincident locations around the periphery of the White Glacier catchment and are independent in that sense. Within the residual domain of WG (Figure 6) we can compare the two DEMs in a different way, for here the TG elevations are copies of the WG elevations and differences in slope are due to differences in the resolution of the DEMs. For two coincident
estimates of slope, the 100 × 100 m computation cell of WG is enclosed by the 200 × 200 m computation cell of TG, so they do not actually share any elevations. In Table 13 it appears that the rms difference of slope is the same, 1.8°, for measurements of different resolution from one map (WG) as it is for measurements from two maps of different resolution (WG and TG; Table 5).

Table 13 also compares WG to CDED and GLOBE within the residual WG domain. The results are consistent with earlier findings (Table 11). The rms difference reaches 5° between WG and CDED and 9° between WG and GLOBE. As in Table 11, there is a pattern in which the mean slope difference increases, without formal statistical significance, as the scale of the parent map decreases. The WG DEM is on average 0.4° steeper than TG and 2.4° steeper than GLOBE. The WG-TG and WG-CDED differences are illustrated in Figure 20, in which panels a and c are analogous to Figure 17b while panels b and d are analogous to Figure 17d. The various histograms show consistently that steeper slopes are found on maps of finer resolution, and that the dispersion of slope differences increases with the difference in scale of the source maps.
### Table 13 — Between-DEM Slope Differences in the WG Domain

<table>
<thead>
<tr>
<th>Difference</th>
<th>WG-TG</th>
<th>WG-CDED</th>
<th>WG-GLOBE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>3820</td>
<td>3843</td>
<td>3843</td>
</tr>
<tr>
<td>Minimum</td>
<td>−11.23</td>
<td>−13.00</td>
<td>−29.16</td>
</tr>
<tr>
<td>Mean</td>
<td>0.44 ± 1.11</td>
<td>1.61 ± 3.64</td>
<td>2.42 ± 6.66</td>
</tr>
<tr>
<td>Rms</td>
<td>1.83</td>
<td>5.13</td>
<td>8.79</td>
</tr>
<tr>
<td>Maximum</td>
<td>13.10</td>
<td>25.86</td>
<td>31.78</td>
</tr>
</tbody>
</table>

Differences (deg) are between slopes calculated from coincident elevation estimates from the four DEMs. In the WG domain, the TG DEM simply samples the WG DEM at reduced resolution (100 m instead of 50 m). CDED and GLOBE are independent of WG. See also Figure 20.

**Summary**

The CDED DEM, based on 1:250 000 scale maps with a contour interval of ∼150 m, and the GLOBE DEM, based on 1:1 000 000 scale maps with a contour interval of ∼300 m, are both more generalized than the Expedition Fiord DEMs. Their elevations are in broad agreement with those of the higher-resolution DEMs, although artefacts related to the contour interval are striking in the hypsometric curves of CDED and especially GLOBE. Slopes are noticeably less on average, and the distribution of slopes shows a relative excess of gentle slopes and a deficit of steep slopes.

Nearest-neighbour resampling is found to be preferable to cubic-convolution resampling.

Making a plausible assumption about the total error of elevations in the TG DEM, comparisons between DEMs show that the total error in CDED elevations is about ±90 m and the total error in GLOBE elevations is about ±160 m. Following analogous reasoning, errors in slopes may be estimated as about ±5° for CDED and ±7-9° for GLOBE. Both CDED and GLOBE give slope estimates which are biased low. The contour-interval artefacts in CDED and GLOBE, and their analogues in the Expedition Fiord DEMs, are responsible for only modest proportions of the map-reading error.
CONCLUSION

Comparisons

The description of DEM errors has evolved rapidly in recent years (e.g., Hunter and Goodchild 1995, 1997; Ehlschlaeger 2002). However there are still few published empirical estimates of the accuracy of DEMs derived from cartographic sources, although photogrammetrically-derived DEMs have been well studied. For example, Davis et al. (2001) reported elevation errors of 2-3 m, relative to GPS elevations with decimetre-level accuracy, for DEMs with horizontal resolution of 1 m and 3 m. Their DEMs were derived from aerial photographs scanned at high resolution and processed digitally. Isaacson and Ripple (1990) compared DEMs from 1:24 000 and 1:250 000 scale maps of a USGS quadrangle in the Cascade Range of Oregon. In a sample of 80 coincident elevations they found a standard error of regression (crudely comparable to our rms differences) of 30.6 m. In a rare glaciological study, Liu and Jezek (1999; see also Liu et al. 1999) compared DEMs from 1:250 000 and 1:50 000 scale maps of the Dry Valley region of Antarctica and found a mean difference of 75 ± 218 m and an rms difference of 132 m, larger than for the analogous comparison (TG-CDED) reported here but roughly similar in units of the contour interval of the 1:250 000 scale maps (200 m in Liu et al.’s study). Much better accuracy is available with modern methods. Bamber et al. (2001) showed that elevations estimated by satellite radar altimetry of the Greenland Ice Sheet had standard errors of about ±2-14 m for slopes less than 1°; these errors are relative to airborne laser altimetry, for which errors are at the sub-metre level.

The mismatch between modern, highly accurate estimates of elevation and the less accurate elevations obtained from old maps is of concern for the analysis of glaciological changes. It means that errors in estimates of volumetric change will be due largely to the old maps. One possible answer would be to return to the old aerial photographs and to generate new maps from them with the advantage of more accurate modern ground control and photogrammetric methods. The effort required would, however, be very substantial.

Confidence Intervals for Elevation Errors

The errors discussed here are not quantified in terms of confidence intervals. Most of our emphasis has been on rms differences which, if certain conventional assumptions are satisfied, represent the standard error of elevation in whichever of the compared DEMs is not regarded as the truth. As such, the rms difference defines a ∼ 68% confidence interval for the elevations of the untruthful DEM. The assumptions are i) that each DEM consists of independent random samples which ii) are all drawn from the same probability distribution and iii) that together the DEMs have a bivariate normal distribution. We have presented some limited indirect evidence for the violation of these assumptions in the DEMs we have studied, but more work would be needed to assess how serious the violations are. Deviations from normality (e.g. Figure 4) and correlation of random errors (e.g. Figure 5) will both tend to reduce statistical confidence somewhat.

If a statement of probability is wanted for elevation errors, it will be necessary either to investigate quantitatively the effect of the violation of assumptions, perhaps in a Monte Carlo experiment; or, as a less time-consuming expedient, to multiply the error estimate by a safety factor. The idea of the safety factor is that, if it is somewhat larger than 1.0, it will correct roughly for the known but unquantified tendency of the violated assumptions to lead to over-optimistic error estimates (confidence intervals which are too narrow). Suppose that a 95% confidence interval is wanted for elevations in the TG DEM, and that the error estimate γT = 15 m is adopted from earlier discussion. If we are willing to believe that 1.2, say, would be a prudent guess for the safety factor, then our 95% confidence interval is $1.2 \times 2 \times 15 \rightarrow \pm 36$ m. The factor of 2 converts the “one-sigma” estimate γT to a “two-sigma” estimate.

The most confidence-sapping of the problems identified in our analysis is probably the persistence of photogrammetric and other blunders (e.g. Figure 11). No amount of proofreading can eliminate such blunders, and the only way to detect them is to sample the topography more than once (hoping that the same blunder will not be made in all of the replicate samples). We are not completely ignorant about these blunders. For example we know that the photogrammetric blunders are more likely to be found above the
snow line, in clumps. We also have crude estimates of their frequency based on small samples; in discussion of Figures 11a and 12a, it was suggested that blunders might be as frequent as one elevation estimate in every forty. The impact of such frequent blunders can be appreciated by comparing the rows of Table 6. It might be acceptable to be guided by the smaller errors in the second row if one could be confident that one’s study region were free of blunders. In general, however, and in glaciological investigations in particular, this will not be so and it will be necessary to accept that undetected blunders are responsible for increasing the uncertainty by 50% or more. Neither of the rows of Table 6 is a satisfactory presentation of the real error structure. Ignoring blunders (second row) makes the presentation incomplete. Including them (first row) inflates the error estimates for the great majority of elevations which are likely to be free of blunders, but does not give a realistic description of the uncertainties in the contaminated elevations.

**Synthesis**

Figure 21 summarizes the total errors \( \gamma \) estimated for the five DEMs in earlier sections. Evidently they can be explained rather well as a function of the contour interval \( c \). The expression \( \gamma = -4.9 + 0.51c \) accounts for 88% of the observed variation. The EF errors seem to be relatively small. It may be that the small sample size of the TG-EF comparison or the close relationship between the two maps is responsible for an underestimate of these errors. If we exclude them, we obtain the best-fitting expression \( \gamma = 3.4 + 0.52c \), explaining 98% of the variance of the remaining four DEMs. It matters little in practical terms which of these expressions we adopt, for they both amount (see figure caption) to the statement that the total error is one half of the contour interval of the parent map.

![Figure 21. Total DEM elevation errors as a function of the contour interval of the parent map. For WG, TG and EF, errors are shown for both of the assumptions of Appendix A: that total error is equal to the map-reading error (upward open triangles), and that total error is the map-reading error plus all of the mapping error identified in a between-map comparison (downward solid triangles). Dashed line: \( \gamma = a + b \cdot c \); the intercept \( a \) is \(-4.9 \pm 9.9\) m and \( b = 0.510 \pm 0.150 \) is the slope. Solid line: a similar expression obtained when the EF errors are excluded; here \( a = 3.4 \pm 4.8 \) m and \( b = 0.520 \pm 0.069 \). Cross: a comparable estimate of DEM error from Liu et al. (1999).](image-url)

Appendix A suggests that the mapping errors of a pair of compared maps may be distributed equably or may be due entirely to one of the two. The latter “lumped-error” assumption implies that the other map is the “truth”. Because the map-reading errors are small by comparison with the mapping errors, these two assumptions lead to inconveniently large differences in the estimated total error. For example for the WG
DEM the total elevation error might be as small as 1 m (the map-reading error) or as large as 19 m (if all of the rms difference between WG and TG elevations is due to mapping errors in the WG map). Although we expect the larger-scale member of any pair of compared maps to be the more accurate, there are in general no grounds for assuming that it is the truth. Unfortunately, the map-comparison method gives no objective guidance as to how to partition the mapping error. The equable-error and lumped-error assumptions are only two among the range of possibilities.

As a working solution, the rule of thumb from Figure 21, namely $\gamma = c/2$, may be acceptable for some purposes although it would be inconsistent with the map comparisons. For example, if the total error of the WG DEM is taken as $\gamma_W = c_W / 2 = 5$ m, Tables 5 and 6 and equations A5 and A6 imply that the total error of the TG DEM is 19.1 m rather than $c_T / 2 = 12.5$ m. Whether this inconsistency is of practical significance will depend on the context. It is instructive to note, however, that if we ignore gross errors this calculation is much closer to consistency with Figure 21: working from the second row of Table 6 instead of the first, $\gamma_T$ is estimated as 11.3 m.

### Table 14 — Candidate Estimates of DEM Elevation Error

<table>
<thead>
<tr>
<th>DEM</th>
<th>$r$</th>
<th>$\gamma$(equable)</th>
<th>$\gamma$(lumped)</th>
<th>$c/2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>WG</td>
<td>1</td>
<td>13</td>
<td>19</td>
<td>5</td>
</tr>
<tr>
<td>TG</td>
<td>6</td>
<td>15</td>
<td>20</td>
<td>12.5</td>
</tr>
<tr>
<td>EF</td>
<td>13</td>
<td>19</td>
<td>24</td>
<td>50</td>
</tr>
<tr>
<td>CDDE</td>
<td>–</td>
<td>90</td>
<td>76</td>
<td></td>
</tr>
<tr>
<td>GLOBE</td>
<td>–</td>
<td>158</td>
<td>152.5</td>
<td></td>
</tr>
</tbody>
</table>

The various estimates are of total error in m, obtained from Appendix A or Figure 21. $r$: total error equal to map-reading error; the parent map is exact. Not available for CDDE, GLOBE. $\gamma$(equable): mapping error shared equally with a comparison DEM. $\gamma$(lumped): mapping error of a comparison DEM deemed to be zero. $c/2$: one half of the contour interval of the parent map. TG: part of TG DEM obtained from TG map; EF: part of TG DEM obtained from EF map.

It would be interesting to know whether there is any deeper reason why one half of the contour interval should be a practical estimate of map and DEM elevation error. It is an estimate which is often seen, but it is usually clear from the context that authors have adopted it casually and as a convenience. Figure 21 represents moderately strong empirical justification, but there is no obvious starting point for a theoretical justification. The violation of statistical assumptions and the probable presence of blunders are roadblocks to further reasoning at present.

A study which is relevant here is that of Shearer (1990), who describes a comparison of DEM elevations derived from maps at scales of 1:10 000, 1:25 000 and 1:50 000, all with a contour interval of 10 m. Comparison errors $\Gamma$, with the 1:10 000 scale map as reference, were 0.09c for the 1:25 000 scale map and 0.29c for the 1:50 000 scale map. Thus, as expected, the contour interval is not the only determinant of elevation error. Parent-map scale also has an effects, in this instance through the digitizing of contours at similar geographical intervals such that DEM interpolation relied on ten times as many points from the 1:10 000 map as from the 1:50 000 map.

Table 14 is a round-number distillation of error estimates from earlier tables for the various DEMs. For each DEM, map-reading error $r$ is a best-case and $\gamma$(lumped) a worst-case estimate. Considering the uncertainty in the other estimates of uncertainty (and ignoring the question whether the EF errors have been underestimated), the rightmost column, $c/2$, offers what is probably the simplest choice among these alternatives for working purposes.
Slope Errors

The structure of the errors estimated for terrain slope is quite different from that for elevation. Between-reader differences are available only for the TG DEM. They range from 0.4° to 0.9° and are thus about one-quarter to one-half as large as differences between WG and TG slopes. Map reading thus seems to be somewhat more uncertain for slope than for elevation. This is consistent with the greater demand for information made of the DEM by a slope estimate.

Maps of larger scale are consistently found to have steeper slopes. Mean differences between WG and TG are less than 0.5° in magnitude, for example, while GLOBE is less steep than WG and TG by 2° or more. Rms differences are greater in magnitude than mean differences, a finding which may be understood in terms of the differences between slope frequency distributions shown in Figures 18, 19 and 20. The phenomenon is traceable to map generalization: on maps of smaller scale, more and more details of terrain structure are left out, and contours become increasingly inadequate representations of the real detail while interpolation is incapable of re-creating it. To know why slopes are underestimated is not, however, the same thing as being able to correct them. Suppose, for example, that in Figure 20c we take the WG slope histogram as the truth for purposes of illustration. Increasing all CDED slopes by the mean slope difference between WG and CDED, 1.6°, will aggravate the problem, yielding a less rather than a more realistic CDED distribution. The correction of slope estimates is a subject for future investigation.

Although this analysis of DEM uncertainties is thus open-ended, it remains true that the large-scale maps of the Expedition Fiord area constitute a valuable glaciological resource for a remote region, fixing with fair accuracy the state of glaciers at known dates in the middle of the 20th century. The newly-available digital versions of these maps, described here, will be essential in comparisons of the 1960 state of glaciation with contemporary and future states recorded in satellite imagery obtained by, for example, the ETM+ instrument aboard Landsat 7 and the Aster instrument aboard Terra. It may also be possible to use them in assessments of change made by laser altimetry. Apart from the growth and shrinkage of glaciers, the DEMs also have applications in the geocoding and radiometric correction of airborne and satellite imagery at both microwave and optical wavelengths. Knowing the uncertainty in elevation and slope will assist materially in appraising the uncertainty remaining after these corrections.

Acknowledgements

We thank Reiner Jung for assistance with the digital elevation model of White Glacier; Candice Stuart, Mathew Laing and Steve Perry for doing most of the work on the digital elevation model of the Thompson Glacier region, and the Government of Ontario for supporting them financially through OWSP, the Ontario Work Study Programme; Miles Ecclestone for help in innumerable ways, including some of the drudgery; McGill University for maintaining the McGill Arctic Research Station over many years and allowing personnel from Trent to stay and work there; Wayne Pollard of McGill University for his leading role in keeping the station open; Trent University for financial support; the Polar Continental Shelf Project, Natural Resources Canada, for essential logistic support; and the CRYSYS programme and M. Sharp of the University of Alberta for supplying the CDED DEM.
Appendix A — Mapping and Map-reading Errors

Suppose that at a point whose location is known exactly the true elevation is $H$, and that

$$H_a = H + B_a \pm \delta_a$$  \hspace{1cm} (A1a)

$$H_b = H + B_b \pm \delta_b$$  \hspace{1cm} (A1b)

are two estimates of $H$ found on different maps, $a$ and $b$. $B_a$ and $B_b$ are unknown biases and are assumed to be zero. $\delta_a$ and $\delta_b$ are random error terms of mean zero and variances equal to $m_a^2$ and $m_b^2$ respectively. $m_a$ and $m_b$ are the expected values of the mapping error for each map. Like $H$, $H_a$ and $H_b$ are unknown; we have to read the maps in order to obtain numbers $h_a$ and $h_b$ with which we can work. This gives rise to the map-reading errors $r_a$ and $r_b$, as follows:

$$h_a = H_a \pm \epsilon_a$$  \hspace{1cm} (A2a)

$$h_b = H_b \pm \epsilon_b$$  \hspace{1cm} (A2b)

where $\epsilon_a$ and $\epsilon_b$ are random error terms of mean zero and variances equal to $r_a^2$ and $r_b^2$ respectively. From (A1) and (A2), a measure of the total error is

$$h_a - h_b = \pm \delta_a \pm \delta_b \pm \epsilon_a \pm \epsilon_b.$$  \hspace{1cm} (A3)

We can now identify the expected value $\Gamma_{ab}$ of $h_a - h_b$ with a between-map rms elevation difference such as one of those in Table 5, and $r_a$, $r_b$ with between-reader rms differences such as those in Tables 1 and 4. $\Gamma_{ab}$ is called the “comparison error”. Assuming that the various errors are independent,

$$\Gamma_{ab} = \sqrt{m_a^2 + m_b^2 + r_a^2 + r_b^2}.$$  \hspace{1cm} (A4)

Separating known terms from unknown, (A4) yields the geometric sum of the two mapping errors

$$m^2 = m_a^2 + m_b^2 = \Gamma_{ab}^2 - r_a^2 - r_b^2,$$  \hspace{1cm} (A5)

We cannot go further than this without some assumption about the relationship between the two components of $m$. Two assumptions which might or might not be realistic in any particular context are

1) lumped error: one mapping error is zero and the other is equal to $m$;

2) equably-distributed error: $m_a$ and $m_b$ are equal and each map has a mapping error of $m/\sqrt{2}$.

Given such an assumption, the total errors in the two maps are

$$\gamma_a = \sqrt{m_a^2 + r_a^2},$$  \hspace{1cm} (A6a)

$$\gamma_b = \sqrt{m_b^2 + r_b^2}.$$  \hspace{1cm} (A6b)

(Interpreted narrowly, the concept of mapping error developed here is absurd. The map cannot yield information without being read. The separation of the two kinds of error is thus artificial, but it is not illusory. The mapping error characterizes errors in the placement of contours on the map; we can only learn about it indirectly, by comparing one map with another. The map-reading error is more aptly named, and characterizes errors in extrapolating from given contour information; we can learn about it by replicate sampling on one map at a time.)

To summarize, if we have coincident replicate samples from two maps we can characterize their elevation and slope errors in terms of:

- map-reading error $r$ for each map;
- a measure $\Gamma$ of comparison error, composed of the two map-reading errors and a pooled estimate of mapping error;
- mapping error $m$ for each map, if we are willing to guess about the relative accuracy of the two;
- total error $\gamma$ in each map, by combining the first and third components above.
Appendix B — Availability and Format

The WG DEM and the TG DEM may be downloaded from the World Wide Web site

HTTP://WWW.TRENTU.CA/GEOGRAPHY/GLACIOLOGY.HTM

at which a digital copy of this Technical Note may also be found. (Printed copies are not available.)

Each of the DEMs is accompanied by a readme file and a file containing sample Fortran reader routines. The WG DEM is in two, and the TG DEM in three, flat binary files. File attributes are given in Table B1.

<table>
<thead>
<tr>
<th>File</th>
<th>Contents</th>
<th>Bytes/element</th>
<th>Columns</th>
<th>Rows</th>
</tr>
</thead>
<tbody>
<tr>
<td>WGTOPO.RS2</td>
<td>Elevation data</td>
<td>2</td>
<td>200</td>
<td>280</td>
</tr>
<tr>
<td>WGLCOD.RS1</td>
<td>Landcode data</td>
<td>1</td>
<td>200</td>
<td>280</td>
</tr>
<tr>
<td>TGTOPO.RS2</td>
<td>Elevation data</td>
<td>2</td>
<td>330</td>
<td>440</td>
</tr>
<tr>
<td>TGLCOD.RS1</td>
<td>Landcode data</td>
<td>1</td>
<td>330</td>
<td>440</td>
</tr>
<tr>
<td>TGICOD.RS1</td>
<td>Icecode data</td>
<td>1</td>
<td>330</td>
<td>440</td>
</tr>
</tbody>
</table>

Elevations (in metres) are 2-byte integers, with −99 denoting missing elevations. The DEM elements are stored in their files in x-axis-major order, by row from south to north with elements running from west to east within each row. Thus the first element in the file is the southwesternmost and the last element is the northeasternmost.

Landcodes and icecodes are single bytes stored in x-axis-major order. The landcode and icecode bytes contain information which is packed as described in Table B2 and Table B3.

Table B2 — Landcode and Icecode Format

<table>
<thead>
<tr>
<th>Bits</th>
<th>Contents</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-1</td>
<td>Land code</td>
</tr>
<tr>
<td>2-5</td>
<td>Basin/ice surface code</td>
</tr>
<tr>
<td>6</td>
<td>Domain code</td>
</tr>
<tr>
<td>7</td>
<td>Missing code</td>
</tr>
</tbody>
</table>

The landcode files have basin codes in bits 2-5 while the icecode file has ice surface codes. The spatial distribution of the basin codes may be understood from Figure 6, and the land codes and ice surface codes are illustrated in Figure 7.

The domain code is 1 for elements within the Expedition Fiord glacier complex (Figure 6), and 0 otherwise. When the domain code is 0, the basin code has the value 15.

The missing code is 1 for elements without a corresponding elevation, and 0 otherwise. When the missing code is 1, the land code and ice surface code are undefined and the basin code has the value 15.
<table>
<thead>
<tr>
<th>Value</th>
<th>Land code</th>
<th>Basin code</th>
<th>Ice surface code</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Cliff</td>
<td>Reserved</td>
<td>No cover, or not ice</td>
</tr>
<tr>
<td>1</td>
<td>Land</td>
<td>Thompson Glacier</td>
<td>Crevasses</td>
</tr>
<tr>
<td>2</td>
<td>Water</td>
<td>Astro Glacier</td>
<td>Morainic cover</td>
</tr>
<tr>
<td>3</td>
<td>Ice</td>
<td>Bellevue Glacier</td>
<td>Crevasses and moraine</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>Finger Glacier</td>
<td>Fine debris cover</td>
</tr>
<tr>
<td>5</td>
<td>Hidden Ice Field</td>
<td></td>
<td>Crevasses and fine debris</td>
</tr>
<tr>
<td>6</td>
<td>Little Phantom Glacier</td>
<td>Reserved</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Phantom Glacier</td>
<td>Reserved</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Transit Glacier</td>
<td>Reserved</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>White Glacier</td>
<td>Reserved</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Wreck Glacier</td>
<td>Reserved</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Crusoe Glacier</td>
<td>Reserved</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Wolf River Glacier</td>
<td>Reserved</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Expedition Fiord</td>
<td>Reserved</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>Reserved</td>
<td>Reserved</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>Missing</td>
<td>Reserved</td>
<td></td>
</tr>
</tbody>
</table>
REFERENCES


Arnold, K.C., 1981, Ice ablation measured by stakes and terrestrial photogrammetry – a comparison on the lower part of the White Glacier, Axel Heiberg Island, Canadian Arctic Archipelago, *Glaciology No. 2, Axel Heiberg Island Research Reports*, McGill University, Montreal. 98p.+maps. (Also Paper 19, National Hydrology Research Institute, Saskatoon.)


Briggs, P.R., 1989, What’s Wrong with this Picture?: Characterization of the N.G.D.C. ETOPO5 5′ × 5′ Global Elevation Dataset using Simple Numerical and Graphic Techniques. Internal report, Department of Geography, Trent University, Peterborough. 45p.


Digital Elevation Models of Axel Heiberg Island Glaciers

Cogley, J.G., 1999a, Axel Heiberg Island: Selected References on Glaciology, Trent Technical Note 99-2, Department of Geography, Trent University, Peterborough. 9p.

Cogley, J.G., 1999b, Photogrammetric Rectification of Oblique Trimetrogon Imagery, Trent Technical Note 99-1, Department of Geography, Trent University, Peterborough. 9p.


Digital Elevation Models of Axel Heiberg Island Glaciers


Müller, F., 1963a, An arctic research expedition and its reliance on large-scale maps, Canadian Surveyor, 17(2), 96-112.

Müller, F., 1963b, Cartas a gran escala para las investigaciones glaciológicas en el Ártico boreal canadiense, Revista Cartográfica, 12(12), 315-324.


National Research Council, 1965a, White Glacier, Axel Heiberg Island, Canadian Arctic Archipelago. Map at 1:10 000 scale in two sheets. Photogrammetric Research Section, National Research Council of Canada, Ottawa, in conjunction with Axel Heiberg Island Expedition, McGill University, Montreal.

Digital Elevation Models of Axel Heiberg Island Glaciers


Ommannay, C.S.L., 1987, Axel Heiberg Island bibliography, in Occasional Paper 12, 5-55, Department of Geography, Trent University, Peterborough, Canada. (Also Miscellaneous Paper 2, Axel Heiberg Island Research Reports, McGill University, Montreal.)

